From 0 to 2018.

Submission deadline: December 31^{st} 2018

Let f be a positive valued continuous function on the interval $[0, 2018]$. Evaluate

$$\int_0^{2018} \frac{f(x)}{f(x) + f(2018 - x)} dx$$

The problem was solved by

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Discussion

Let $I = \int_0^{2018} f(x)/(f(x)+f(2018-x))dx$. Then

$$I = \int_0^{2018} 1 - \frac{f(2018-x)}{f(x)+f(2018-x)}dx$$

Thus,

$$I = 2018 - \int_0^{2018} \frac{f(2018-x)}{f(x)+f(2018-x)}dx \tag{1}$$

Using the change of variables $t = 2018 - x$, it is easy to see that

$$\int_0^{2018} \frac{f(x)}{f(x)+f(2018-x)}dx = \int_0^{2018} \frac{f(2018-t)}{f(2018-t)+f(t)}dt$$

Now, from equation (1) we get

$$I = 2018 - I$$

Therefore

$$\int_0^{2018} \frac{f(x)}{f(x)+f(2018-x)}dx = \frac{2018}{2}$$