An Alternating Series

Submission deadline: November 30th 2018

Given that

\[ 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \cdots = \frac{\pi^2}{6} \]

find

\[ 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \frac{1}{6^2} + \cdots \]

The problem was solved by

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Discussion:
It is known that
\[ 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \cdots = \frac{\pi^2}{6} \]  
(1)

Multiplying equation (1) by 1/2² yields that
\[ \frac{1}{2^2} + \frac{1}{(2 \cdot 2)^2} + \frac{1}{(2 \cdot 3)^2} + \frac{1}{(2 \cdot 4)^2} + \frac{1}{(2 \cdot 5)^2} + \frac{1}{(2 \cdot 6)^2} + \cdots = \frac{1}{2^4} \frac{\pi^2}{6} \]  
(2)

Now, subtracting two times equation (2) from equation (1) results in
\[ 1 - \frac{1}{2^2} - \frac{1}{3^2} - \frac{1}{4^2} - \frac{1}{5^2} - \frac{1}{6^2} + \cdots = \frac{\pi^2}{6} - \frac{1}{2^2} \frac{\pi^2}{6} \]
\[ = \frac{\pi^2}{12} \]

Also see solution to February 2018 problem.