They aren't Twin Primes

Submission deadline: August 31st 2019

Prove that if one of the numbers 2^n-1 and 2^n+1 is prime, where $n > 2$, then the other number is composite.

The problem was solved by

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Discussion:

Since $(2^n - 1)(2^n + 1) = 4^n - 1$, we have that

$$(2^n - 1)(2^n + 1) = (4 - 1)(4^{n-1} + \cdots + 1)$$

Thus, 3 divides the product of $(2^n - 1)$ and $(2^n + 1)$, but one of them is a prime larger than 3, therefore, the other must be divisible by 3.

There are many other solutions as well. One of them is to look at the three consecutive numbers $2^n - 1, 2^n, 2^n + 1$.

Another is to analyse the two numbers when n is odd and when n is even separately.