2019!

Submission deadline: March 31^{st} 2019

Evaluate

$$1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \dots + 2019 \cdot 2019!$$

The problem was solved by

- Ievgen Murzak, University of Houstan, USA.
- Ruben Victor Cohen, Argentina.
- Shubhan Bhatia, Grade 11, Gems Modern Academy, Dubai, UAE.
- Alfaisal A. Hasan, PSA, Sharjah, UAE.
- Hichem Zakaria Aichour.

Discussion

Let
$$a_p = p \cdot p!$$
 Then $a_p = ((p+1) - 1) \cdot p!$ Therefore

$$a_p = (p+1)! - p!$$

Thus

$$\sum_{p=1}^{2019} a_p = (2! - 1!) + (3! - 2!) + \dots + (2020! - 2019!)$$

Therefore

$$1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \dots + 2019 \cdot 2019! = 2020! - 1.$$