Erase the Last

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Find all positive integers n for which the number obtained by erasing the last digit is a divisor of n.

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Discussion

Let $n = a_1a_2\cdots a_{m-1}a_m$ where $0 \leq a_i \leq 9$. If p is the integer obtained by erasing the last digit of n then $p = a_1a_2\cdots a_{m-1}$. Therefore $n = 10p+a_m$. Thus,

$$\frac{n}{p} = 10 + \frac{a_m}{p}$$

It is clear that p divides n if and only if p divides a_m.

If $a_m = 0$, then p can be any integer. Thus n can be any natural number that ends in 0.

If $a_m = 1$, then $p = 1$, hence $n = 11$.
If $a_m = 2$, then $p = 1, 2$ hence n is 12 or 22.
If $a_m = 3$, then $p = 1, 3$ hence n is 13 or 33.
If $a_m = 4$, then $p = 1, 2, 4$ hence n can be $14, 24, 44$.
If $a_m = 5$, then $p = 1, 5$ hence n is 15 or 55.
If $a_m = 6$, then $p = 1, 2, 3, 6$ hence n can be $16, 26, 36, 66$
If $a_m = 7$, then $p = 1, 7$ hence n can be 17 or 77.
If $a_m = 8$, then $p = 1, 2, 4, 8$ hence n can be $18, 28, 48, 88$.
If $a_m = 9$, then $p = 1, 3, 9$ hence n can be $19, 39, 99$.