Opposite Triangles

Submission deadline: April 28^{th} 2021

Let $ABCD$ be a convex quadrilateral and O be the point of intersection of its diagonals. If

$$(\text{Area of } AOD) \cdot (\text{Area of } COB) = 2021$$

find

$$(\text{Area of } ABO) \cdot (\text{Area of } DOC)$$

Note: Vertices A, B, C, D are labelled clockwise.

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Discussion:

Angles $D\hat{O}A$ and $A\hat{O}B$ both cannot be greater than $\pi/2$. Thus assume that $D\hat{O}A \leq \pi/2$, and call it γ.

Now it is easy to see that

Area of $AOD = \frac{1}{2}|OA| \cdot \sin(\gamma) \cdot |OD|$

and

Area of $COB = \frac{1}{2}|OB| \cdot \sin(\gamma) \cdot |OC|$

Similarly we have

Area of $ABO = \frac{1}{2}|OA| \cdot \sin(\gamma) \cdot |OB|$

and

Area of $DOC = \frac{1}{2}|OC| \cdot \sin(\gamma) \cdot |OD|$

Thus,

(Area of AOD) $\cdot$ (Area of COB) $=$ (Area of ABO) $\cdot$ (Area of DOC)