Odd Factorials

Submission deadline: December 28^{th} 2021

Find

$$\frac{1}{1!2020!}+\frac{1}{3!2018!}+\frac{1}{5!2016!}+\cdots+\frac{1}{2017!4!}+\frac{1}{2019!2!}+\frac{1}{2021!}$$

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Discussion;

For $0 \leq k \leq n$, let ${}^{n}C_k = \dfrac{n!}{k! \cdot (n-k)!}$. From Binomial theorem it follows that

$$(1+1)^{2021} = {}^{2021}C_0 + {}^{2021}C_1 + {}^{2021}C_2 + {}^{2021}C_3 + {}^{2021}C_4 + {}^{2021}C_5 + \cdots + {}^{2021}C_{2021}$$

and

$$(1-1)^{2021} = {}^{2021}C_0 - {}^{2021}C_1 + {}^{2021}C_2 - {}^{2021}C_3 + {}^{2021}C_4 - {}^{2021}C_5 + \cdots - {}^{2021}C_{2021}$$

Subtracting the second equation above from the first results in

$$2^{2021} = 2({}^{2021}C_1 + {}^{2021}C_3 + {}^{2021}C_5 + \cdots + {}^{2021}C_{2021})$$

Thus, $\dfrac{1}{1!2020!} + \dfrac{1}{3!2018!} + \dfrac{1}{5!2016!} + \cdots + \dfrac{1}{2017!4!} + \dfrac{1}{2019!2!} + \dfrac{1}{2021!}$ is equal to

$$\frac{2^{2020}}{2021!}$$