Symmetry.

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Find all positive integers a, b and c such that

$$ab + bc + ac > abc$$

The problem was solved by

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Discussion

Let

$$x = ab + bc + ac - abc$$

First assume that $a \leq b \leq c$.

Then, $ab + bc + ac \leq 3bc$. Therefore, if $a \geq 3$, then the given inequality cannot be satisfied, hence $a = 2$ or $a = 1$.

Now let $a = 2$.

Then,

$$x = 2b + c(2 - b)$$

It is clear that for x to be positive, b must be either 3 or 2. If $b = 3$, then it is clear that $c \leq 5$.

Thus we get one set of solutions written as triples; (2,3,5), (2,3,4) and (2,3,3).

If $b = 2$, then

$$x = 2b$$

hence we get another set of solutions $(2, 2, c)$ where c is an integer not less than 2.

If $a = 1$, then

$$x = b + c$$

and we get the solutions $(1, b, c)$, where b and c are positive integers.

Since the given expression is symmetric we can take any permutation of the solutions above.