Logs

Submission deadline: February 28th 2022

Find
\[
\frac{1}{\log_2(2022!)} + \frac{1}{\log_3(2022!)} + \cdots + \frac{1}{\log_{2022}(2022!)}
\]

The problem was solved by

- Muhammed YÜKSEL, Hacettepe University Automotive Eng-Mech. Eng, Ankara/Turkey
- Gurkan Koray Akpınar, Aydin, Turkey.
- Hari Kishan, D.N. College, Meerut, India.
- Rohan Mitra, American University of Sharjah, UAE.
- Atakan Erdem, Middle East Technical University, Ankara, Turkey.
Discussion:

Since \( \frac{1}{\log_m(2022!)} = \log_{2022!}(m) \), it is easy to see that the given series is equal to

\[
\log_{2022!}(2) + \log_{2022!}(3) + \cdots + \log_{2022!}(2022)
\]

The sum above is clearly equal to \( \log_{2022!}(2 \cdot 3 \cdots 2022) \), which is equal to 1.