

Squares and Exponents

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Find all integer solutions of

$$2^x = x^2$$

The problem was solved by

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Discussion;

Since x^2 is even if and only if x is even it is clear that $x = 2p$. Thus $2^{2p} = 4p^2$. Therefore,

$$2^{p-1} = p \tag{1}$$

If $p > 2$, by looking at the binomial expansion of $(1 + 1)^{p-1}$, it can be concluded that $2^{p-1} > p$. Thus $p \leq 2$.

It is easy to see that $p = 1$, and $p = 2$, are solutions of (1). This results in $x = 2$ and $x = 4$ which are solutions of the given equation. Since p can take only two values, $x = 2$ and $x = 4$ are the only solutions.