

Polynomial Factoring

Submission deadline: November 28th 2021

Find all complex valued constants a, b, c, A, B and C such that

$$x^2 + y^2 + z^2 = (ax + by + cz)(Ax + By + Cz)$$

for all x, y, z .

The problem was solved by

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Discussion;

Assume that $x^2 + y^2 + z^2 = (ax + by + cz)(Ax + By + Cz)$. Then it easily follows that $aA = 1, bB = 1$ and $cC = 1$. Thus $A/B = b/a$.

The coefficient of xy term is $aB + bA$ thus $aB + bA = 0$. Therefore, $-a/b = A/B$ and now it follows that $a^2 + b^2 = 0$. Through similar arguments it can be seen that $a^2 + c^2 = 0$, and $b^2 + c^2 = 0$. Hence $b^2 = 0$. Clearly, b is non-zero, hence it can be concluded that such constants do not exist.