Symmetry.

Submission deadline: October 30^{th} 2020

Let F be a differentiable function on $[0, 2020]$ such that $F'(2020 - x) = F'(x)$ for all x in the domain. Evaluate

$$\int_0^{2020} F(x)dx$$

The problem was solved by

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Discussion
Clearly,

$$F(x) - F(0) = \int_0^x F'(t)dt.$$

Therefore

$$F(x) - F(0) = \int_0^x F'(2020 - t)dt.$$

By change of variables, $2020 - t = u$, we get that

$$F(x) - F(0) = -\int_{2020}^{2020-x} F'(u)du.$$

Thus, $F(x) - F(0) = -(F(2020 - x) - F(2020))$ which yields

$$F(x) + F(2020 - x) = F(2020) + F(0).$$

Therefore,

$$\int_0^{2020} F(x)dx + \int_0^{2020} F(2020 - x)dx = 2020(F(2020) + F(0))$$

By change of variables in the second term of the left hand side above, it is easy to see that two terms on the left are equal to each other. Hence,

$$\int_0^{2020} F(x)dx = 2020\frac{F(2020) + F(0)}{2}$$

Note that $2F(1010) = F(2020) + F(0)$.

Many solutions were variations of the one above. However, Dilmini Mannaperuma gave a more "geometrical" solution by taking symmetry into consideration. That solution clearly reveals why the answer looks like the area of a trapezoid.