

Fourteen

Submission deadline: October 28th 2021

Find all integer solutions of

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{14}$$

The problem was solved by

- Rohan Mitra, *American University of Sharjah, UAE.*
- Ferdi, *Hasanuddin University, Indonesia.*
- Parv Bhadra, *Grade 11, GEMS Modern Academy, Dubai, UAE.*
- Gürkan Koray Akpınar, *Turkey.*
- Ruben Victor Cohen, *Argentina.*
- Atakan Erdem, *Middle East Technical University, Ankara, Turkey.*
- Hari Kishan, *(Ex.) Department of Mathematics, D.N. College, Meerut, India.*

Discussion;

It is easy to see that $x = 14\frac{y}{y-14}$ which can be rewritten as

$$x = 14 + \frac{2^2 \cdot 7^2}{y-14}$$

Thus, x is a non-zero integer if and only if, when $y-14$ takes any of the values $\pm 1, \pm 2, \pm 2^2, 2 \cdot 7, \pm 2 \cdot 7^2, \pm 2^2 \cdot 7, \pm 2^2 \cdot 7^2, \pm 7, \pm 7^2$. Written in (x, y) form this results in solutions $(210, 15), (-182, 13), (112, 16), (-84, 12), (63, 18), (-35, 10), (28, 28), (21, 42), (7, -14)$. Another set of solutions can be obtained by swapping the values of x and y .