x and $\dfrac{1}{x}$

Submission deadline: March 30^{th} 2022

Let n be a positive integer and $0 < \alpha < \pi/2$. If $x + \dfrac{1}{x} = 2\cos(\alpha)$, find $x^n + \dfrac{1}{x^n}$

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Discussion.

It is easy to see that $x^2 - 2x\cos(\alpha) + 1 = 0$. Solving the quadratic equation yields that $x = \cos(\alpha) \pm i\sin(\alpha)$. Thus, $x = e^{i\alpha}$ or $x = e^{-i\alpha}$. Either root for x results in $x^n + \dfrac{1}{x^n} = e^{in\alpha} + e^{-in\alpha}$. Hence

$$x^n + \frac{1}{x^n} = 2\cos(n\alpha)$$