Let $n$ be a positive integer and $0 < \alpha < \pi/2$. If $x + \frac{1}{x} = 2\cos(\alpha)$, find $x^n + \frac{1}{x^n}$.

The problem was solved by

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Discussion.

It is easy to see that \( x^2 - 2x \cos(\alpha) + 1 = 0 \). Solving the quadratic equation yields that \( x = \cos(\alpha) \pm i \sin(\alpha) \). Thus, \( x = e^{i\alpha} \) or \( x = e^{-i\alpha} \). Either root for \( x \) results in \( x^n + \frac{1}{x^n} = e^{ina} + e^{-ina} \). Hence

\[
x^n + \frac{1}{x^n} = 2 \cos(n\alpha)
\]