Halve the terms

Submission deadline: April 28^{th} 2023

Prove that for each positive integer n

$$1-\frac{1}{2}+\frac{1}{3}-\cdots+\frac{1}{2n-1}=\frac{1}{n}+\frac{1}{n+1}+\cdots+\frac{1}{2n-1}$$

The problem was solved by

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Discussion:

We use induction. When $n = 1$, it is easy to see that each side of the equation above is equal to one. Now assume that the equation above is true up to $n = p$. Let $n = p + 1$, and

$$S = 1 - \frac{1}{2} + \cdots + \frac{1}{2(p+1) - 1}$$

The last term above is $1/(2p+1)$, thus the series above written with more terms is

$$1 - \frac{1}{2} + \cdots + \frac{1}{2p-1} - \frac{1}{2p} + \frac{1}{2p+1}$$

By induction hypothesis the first $2p - 1$ terms in the series above are equal to $1/p + \cdots + 1/(2p-1)$. Thus,

$$S = \frac{1}{p} + \frac{1}{p+1} + \cdots + \frac{1}{2p-1} - \frac{1}{2p} + \frac{1}{2p+1}$$

Therefore

$$S = \frac{1}{p+1} + \cdots + \frac{1}{2p-1} + \frac{1}{2p} + \frac{1}{2p+1}$$

Since $2p + 1 = 2(p + 1) - 1$, the assertion is true for $n = p + 1$.