Halve the terms

Submission deadline: April 28th 2023

Prove that for each positive integer $n$

$$1 - \frac{1}{2} + \frac{1}{3} - \cdots + \frac{1}{2n-1} = \frac{1}{n} + \frac{1}{n+1} + \cdots + \frac{1}{2n-1}$$

The problem was solved by

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Discussion:
We use induction. When \( n = 1 \), it is easy to see that each side of the equation above is equal to one. Now assume that the equation above is true up to \( n = p \). Let \( n = p + 1 \), and

\[ S = 1 - \frac{1}{2} + \cdots + \frac{1}{2(p+1) - 1} \]

The last term above is \( 1/(2p+1) \), thus the series above written with more terms is

\[ 1 - \frac{1}{2} + \cdots + \frac{1}{2p - 1} - \frac{1}{2p} + \frac{1}{2p + 1} \]

By induction hypothesis the first \( 2p - 1 \) terms in the series above are equal to \( 1/p + \cdots + 1/(2p - 1) \). Thus,

\[ S = \frac{1}{p} + \frac{1}{p+1} + \cdots + \frac{1}{2p - 1} - \frac{1}{2p} + \frac{1}{2p + 1} \]

Therefore

\[ S = \frac{1}{p+1} + \cdots + \frac{1}{2p - 1} + \frac{1}{2p} + \frac{1}{2p + 1} \]

Since \( 2p + 1 = 2(p + 1) - 1 \), the assertion is true for \( n = p + 1 \).