Recurrence

Submission deadline: March 30^{th} 2022

Let a and b be fixed positive integers. Find the general solution of the recurrence relation

$$x_{n+1} = x_n + a + \sqrt{b^2 + 4ax_n},\ n = 0, 1, 2, \cdots$$

where $x_0 = 0$.

The problem was solved by

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Discussion;

A straight forward computation of the first few terms suggest that $x_n = n^2a + nb$.

Now assume that $x_n = n^2a + nb$ for $n = 1, \cdots, k$. Then $x_{k+1} = k^2a + kb + a + \sqrt{b^2 + 4a(k^2a + kb)}$, and further simplification yields that $x_{k+1} = (k^2 + 2k + 1)a + (k + 1)b$. Thus, from the principle of mathematical induction it follows that $x_n = n^2a + nb$ is true for all n.