Recursive

Submission deadline: March 29^{th} 2022

Let f be a function defined on all real numbers such that for all x, we have $f(x+5) \geq f(x)+5$, and $f(x+1) \leq f(x)+1$. If $f(1)=1$, find $f(2022)$.

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Discussion.

Since $f(x+1) - f(x) \leq 1$, we get $\sum_{j=0}^{4} f(x+j+1) - f(x+j) \leq 5$. Thus $f(x+5) \leq f(x) + 5$. Since $f(x+5) \geq f(x) + 5$, it is easy to see that $f(x+5) = f(x) + 5$. If $f(x_0+1) < f(x_0) + 1$, for some x_0, then from an argument similar to the one above it follows that $f(x_0+5) < f(x_0) + 5$. Thus we have $f(x+1) = f(x) + 1$ for all x. Since $f(1) = 1$, we get that $f(2) = 2, f(3) = 3, \cdots$ and thus $f(2022) = 2022$.