Let $f$ be a function defined on all real numbers such that for all $x$, we have $f(x + 5) \geq f(x) + 5$, and $f(x + 1) \leq f(x) + 1$. If $f(1) = 1$, find $f(2022)$.

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Discussion.

Since \( f(x + 1) - f(x) \leq 1 \), we get \( \sum_{j=0}^{4} f(x + j + 1) - f(x + j) \leq 5 \). Thus \( f(x + 5) \leq f(x) + 5 \). Since \( f(x + 5) \geq f(x) + 5 \), it is easy to see that \( f(x + 5) = f(x) + 5 \). If \( f(x_0 + 1) < f(x_0) + 1 \), for some \( x_0 \), then from an argument similar to the one above it follows that \( f(x_0 + 5) < f(x_0) + 5 \). Thus we have \( f(x + 1) = f(x) + 1 \) for all \( x \). Since \( f(1) = 1 \), we get that \( f(2) = 2, f(3) = 3, \ldots \) and thus \( f(2022) = 2022 \).