Remainder

Submission deadline: October 29^{th} 2022

Find the remainder of the division of the polynomial

$$x + x^9 + x^{25} + x^{49} + \cdots + x^{99^2}$$

by $x^3 - x$.

The problem was solved by

- Atakan Erdem, *Middle East Technical University, Ankara, Turkey.*

- Kaustuv Sengupta, *Calcutta, India.*

Discussion:

Let $P(x)$ be the given polynomial. Since the remainder term is at most a degree 2 polynomial we have

$$P(x) = (x^3 - x)Q(x) + (ax^2 + bx + c)$$

Let $x = 0$. Then it can be concluded that $c = 0$. Letting $x = 1$, yields that $50 = a + b$, and $x = -1$, results in $-50 = a - b$. By solving the two equations above we see that $a = 0$ and $b = 50$. Thus the remainder is $50x$.