Mathematical techniques for modelling and numerical simulation of environmental problems

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Mathematical techniques have been widely used in the literature to model and solve many applications in engineering. Partial differential equations (PDEs) are the core in modelling these applications using well-established conservation laws in physics. Nowadays modelling hydraulics in large environmental systems requires access and analysis to tremendous sets of data. In the current work we give a survey on mathematical equations used in the field of modelling and numerical simulation of:

1. Transport and dispersion of pollutant in the North Sea
2. Oceanic recirculation in the strait of Gibraltar
3. Eutrophication in the lake of Bouregreg.

We present detailed formulation of the PDEs governing these models along with condition required for their mathematical analysis and well-posedness. Numerical methods used for the space and time discretization will also covered in this study. The focus is on a class of finite volume and finite element methods which employ unstructured meshes and satisfy high levels of accuracy and efficiency. Preparation of the data for these applications is also a crucial step towards any efficient and accurate modelling of their dynamics. Other mathematical tools include reconstruction of the data for PDEs in their irregular geometries, complex bathymetry, coupled boundary conditions, nonlinear friction terms at the bed and the free-surface, and wind and dispersion coefficients. These reconstructed topographies are tested in high performance computing and both deterministic and stochastic simulations are presented. Three applications in environmental engineering are considered in the current work including numerical simulation of oil pollution in sea-surface flows.