Almost a Square

Submission deadline: July 31^{st} 2019

Evaluate

$$\sum_{k=1}^{2019} \frac{k}{k^4+k^2+1}$$

The problem was solved by

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Discussion
Since $k^4 + k^2 + 1 = (k^2 + 1)^2 - k^2$ we have that

$$\frac{k}{k^4 + k^2 + 1} = \frac{1}{2}\left(\frac{1}{k(k-1)+1} - \frac{1}{k(k+1)+1}\right)$$

Thus

$$\sum_{k=1}^{2019} \frac{k}{k^4 + k^2 + 1} = \frac{1}{2}\left(1 - \frac{1}{2019 \cdot 2020 + 1}\right)$$