Almost a Square

Submission deadline: July 31st 2019

Evaluate

$$\sum_{k=1}^{2019} \frac{k}{k^4 + k^2 + 1}$$

The problem was solved by

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Discussion
Since \( k^4 + k^2 + 1 = (k^2 + 1)^2 - k^2 \) we have that

\[
\frac{k}{k^4 + k^2 + 1} = \frac{1}{2} \left( \frac{1}{k(k - 1) + 1} - \frac{1}{k(k + 1) + 1} \right)
\]

Thus

\[
\sum_{k=1}^{2019} \frac{k}{k^4 + k^2 + 1} = \frac{1}{2} \left( 1 - \frac{1}{2019 \cdot 2020 + 1} \right)
\]