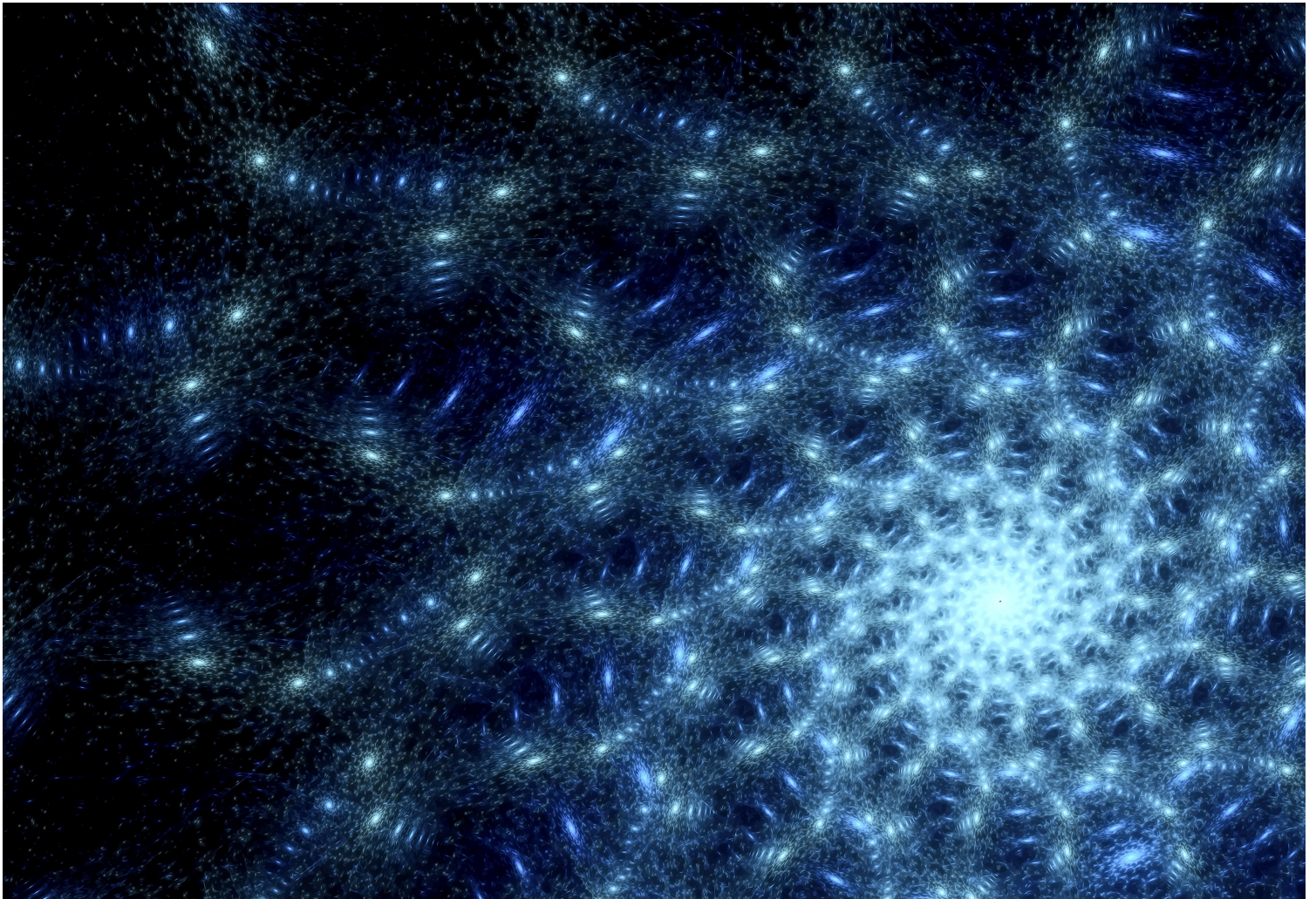


# The Third International Conference on Mathematics and Statistics | AUS-ICMS '20

February 6–9, 2020

**Book of abstracts**



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## WELCOME NOTE

*From the organizing chair*

The Department of Mathematics and Statistics at American University of Sharjah welcomes you to the Third International Conference on Mathematics and Statistics (AUS-ICMS'20). The first two editions of the conference, held at American University of Sharjah in 2010 and 2015, attracted members of the mathematical community from over 45 countries who joined together to present, discuss, promote and disseminate research in every field of mathematics, statistics and their applications. With the same objective, the Third International Conference on Mathematics and Statistics has attracted over 180 mathematicians and statisticians, who join the conference from nearly 43 countries.

In addition to keynote lectures delivered by renowned mathematicians and parallel sessions in all areas of mathematics and statistics, the scientific program of this year's conference includes special sessions in contemporary content-rich topics in algebra, number theory, financial mathematics, mathematics in biology and data mining and statistical learning. It is an honor to see so many distinguished researchers gathered here at the conference to present and discuss the latest developments in these fields.

A special issue of the SCOPUS-indexed journal *Advances in Pure and Applied Mathematics (APAM)* is dedicated to publishing selected papers presented at the conference. The conference program also includes social and professional interaction activities including the banquet dinner at Occidental Sharjah Grand Hotel on Friday night and an excursion to Dubai Global Village on Saturday afternoon. This will give you a great opportunity to make new friends, renew acquaintances and make your stay in Sharjah a truly enjoyable experience.

On behalf of the conference organizing committee, I would like to express my gratitude to American University of Sharjah—under the auspices of His Highness Sheikh Dr. Sultan Bin Muhammad Al Qasimi, Member of the UAE Supreme Council, Ruler of Sharjah and President of American University of Sharjah—for the great support throughout the organization of the conference. I would also like to thank SIAM and IMS for their technical sponsorship of the conference. Sincere thanks and appreciation go to the keynote speakers, special session organizers, members of the international advisory board, session chairs, our sponsors, and AUS staff and student helpers for their diligence and support.

Finally, I wish to thank you for your valuable participation in the conference and contribution to its success. We are confident that you will find the conference stimulating and rewarding.

*Hana Sulieman*

*Head of the Department of Mathematics and Statistics and Associate Dean for Graduate Studies  
College of Arts and Sciences  
American University of Sharjah*

# AUS-ICMS '20 Organizing Committee

## **Conference Chair**

Hana Sulieman

## **Organizing Committee**

Taher Abualrub

Ayman Badawi

Stephen Chan

Abdul Salam Jarrah (*SIAM Representative*)

Amjad Tuffaha

## **International Advisory Board**

Helene Barcelo, USA

Yonglin Cao, China

James Coykendall, USA

Alicia Dickenstein, Argentina

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Youssef Belhamadia

James Griffin

Gajath Gunatillake

Thomas Wunderli

## **Local Arrangements**

Diana Audi

Faruk Uygul

## **Website Coordinator**

Mujo Mesanovic

## **Sponsorship Coordinator**

Saadia Khouyibaba

## Daily Schedule for AUS-ICMS20

Time	Day 1, February 6
8:30–10:30	Registration & Breakfast
10:30–11:00	Opening Ceremony
11:00–12:00	Plenary I
12:00–12:15	Group Photo
12:15–14:00	Lunch
14:00–15:40	Session A1
15:40–16:00	Coffee Break
16:00–18:30	Session A2

Time	Day 2, February 7
8:30– 0:00	Registration
9:00–10:40	Session B1
10:40–11:00	Coffee Break
11:00–12:15	Session B2
12:15–14:00	Lunch
14:00–16:05	Session B2
18:30–22:00	Banquet Dinner Occidental Sharjah Grand Hotel

Time	Day 3, February 8
8:30–10:00	Registration
9:00–10:00	Plenary II
10:00–10:30	Coffee Break & Group Photo
10:30–12:35	Session C1
12:35 – 14:00	Lunch
14:00–16:05	Session C2
16:30–22:00	Excursion to Dubai Global Village ( <i>free of charge</i> )

**AUS-ICMS20**  
**February 6<sup>th</sup> - 9<sup>th</sup>, 2020**

**CONFERENCE PROGRAM**

<b>Thursday 6<sup>th</sup> February, 2020</b>			
<b>Registration &amp; Breakfast</b> <b>8:30 – 10:30</b>			
<b>Opening Ceremony</b> <b>10:30 – 11:00</b>			
<b>Plenary Lecture I</b> <i>Modelling Collective Cell Migration</i> <b>Professor Philip K. Maini</b> <b>University of Oxford</b> <b>11:00 – 12:00</b>			
<b>Group Photo Op</b> <b>12:00 – 12:15</b>			
<b>Lunch</b> <b>12:15-14:00 pm</b>			
<b>Session A1: Applied Mathematics</b> <b>Chair: Abdulhakeem Yusuf</b> <b>Room: SBA0005</b>			
<b>Time</b>	<b>ID</b>	<b>Title</b>	<b>Speaker</b>
14:00-14:25	AM-3003	Hydromagnetic and Thermal Boundary Layer Flow due to Radial Stretching Sheet with Dufour and Soret Effects	Abdulhakeem Yusuf
14:25-14:50	AM-3033	Autoionization of Hollow Atomic Ions	Shahin A. Abdel-Naby
14:50-15:15	AM-3007	CFD – A Tool to Study Air Pollution Problems	Khaleel Ahmed
15:15-15:40	MB-3008	Stochastic Delay Differential Equations for Three Species Prey-Predator System with Allee Effect	Hebatallah Alsakaji
<b>Special Session A1: Rings, Monoids and Module Theory</b> <b>Chair: Sylvia Wiegand</b> <b>Room: SBA0001</b>			
<b>Time</b>	<b>ID</b>	<b>Title</b>	<b>Speaker</b>
14:00-14:25	AA3027	On t-local Domains and Valuation Domains	Marco Fontana
14:25-14:50	AA3003	On Valuation Factorization Domains	Andreas Reinhart
14:50-15:15	AA3010	Class(semi)group of Prufer Domains and Atomicity	Richard Erwin Hasenauer
15:15-15:40	AA3029	The Class Group of h-local Prufer Domains	Gyu Whan Chang

<b>Special Session A1: Discrete Dynamic Modeling of Biological Systems</b> <b>Chair: Abdul Jarrah</b> <b>Room: SBA0006</b>			
Time	ID	Title	Speaker
14:00-14:30	MB-3004	A Multiscale Model of the Innate Immune Response to Respiratory Fungal Infections	Reinhard Laubenbacher
14:30-15:00	MB-3009	Deciphering Yeast Physiology by a Multi-Scale Framework Integrating Cell Cycle and Metabolism	Matteo Barberis
15:00-15:25	MB-3015	Compositionality in the Boolean Model of Regulatory Networks	Marco Pedicini
<b>Session A1: Number Theory</b> <b>Chair: Armen Bagdsaryan</b> <b>Room: SBA0010</b>			
Time	ID	Title	Speaker
14:00-14:25	NT-3006	On Some Relations Involving Zeros of Riemann Zeta Function and other Zeta Related Functions	Armen Bagdsaryan
14:25-14:50	NT-3001	Analytic Expressions of Characters Defined on Witt Vectors Rings	Siham Mokhfi
14:50-15:15	NT-3008	On Certain Multiple Dirichlet Series of Completely Multiplicative Function	Nabil Tahmi
15:15-15:40	NT-3007	On a Method of Summation of Infinite Series with some Applications to Special Numbers and Zeta Functions	Armen Bagdsaryan
<b>Session A1: Statistics</b> <b>Chair: Sana Louhichi</b> <b>Room: SBA0009</b>			
Time	ID	Title	Speaker
14:00-14:25	ST-3003	Nonparametric Estimation for the Hazard Function	Mounir Arfi
14:25-14:50	ST-3007	On Smoothing Parameters Selection Problems in Nonparametric Regression Models with Dependent Errors	Sana Louhichi
14:50-15:15	ST-3015	Smooth Nonparametric Regression under Shape Restrictions	Hongbin Guo
15:15-15:40	ST-3016	Value-At-Risk Prediction through Vine Copula	Arief Hakim
<b>Session A1: Financial Mathematics</b> <b>Chair: Mayank Goel</b> <b>Room: SBA0012</b>			
Time	ID	Title	Speaker
14:00-14:25	FM-3002	Causality and Price Discovery of Cross-Listed Nifty Futures	K. Kiran Kumar
14:25-14:50	FM-3006	Risk-Sensitive Benchmarked Portfolio Optimization with General Non-Negative Economic Factors	Mayank Goel
14:50-15:15	FM-3008	Valuation of Volatility Derivatives under Markov Switching Stochastic Volatility Model	Youssef El Khatib
<b>COFFEE BREAK</b> <b>15:40-16:00 pm</b>			

<b>Special Session A2: Financial Statistics with Application in Cryptocurrency and Blockchain</b> <b>Chair: Stephen Chan</b> <b>Room: SBA0011</b>			
Time	ID	Title	Speaker
16:00-16:25	FM-3007	The Adaptive Market Hypothesis in the High Frequency Cryptocurrency Market	Jeffrey Chu
16:25-16:50	FM-3004	Trading Models on Cryptocurrencies	Shou Hsing Shih
16:50-17:15	ST-3012	Modelling Cryptocurrencies using Undirected Graphs	Paola Stolfi
17:15-17:40	FM-3003	Flexible Models for Stock Returns based on Student's t-Distribution	Emmanuel Afuecheta
<b>Special Session A2: Statistical Learning-Data Mining-Probability</b> <b>Chair: Mahmoud Awad</b> <b>Room: SBA0009</b>			
Time	ID	Title	Speaker
16:00-16:25	ST-3018	Travelers' Perception of Service Quality at Dubai International Airport	Mahmoud Awad
16:25-16:50	ST-3023	An Artificial Neural Network Approach to Nonparametric Control Chart	Nimbale S. M
16:50-17:15	ST-3004	A Hybrid Artificial Neural Networks and Arima Models for Forecasting Electricity Consumption in Palestine	Samir K. Safi
17:15-17:40	MD-3003	Design New Pseudorandom Number Generators using Tabu Programming	Emad Mabrouk
17:40-18:05	ST-3017	Human Gut Microbiota Composition and Functionality Correlation with Age	Mohammad Tahseen Al Bataineh
<b>Session A2: Analysis</b> <b>Chair: Oscar Fonseca</b> <b>Room: SBA0010</b>			
Time	ID	Title	Speaker
16:00-16:25	AN-3014	Approximation Properties of $\Theta$ -Means of Walsh-Fourier Series in Different Spaces	Karoly Nagy
16:25-16:50	AN-3015	Some Fixed Point Results in Complex Valued Dislocated Metric Space and its Applications	Rajinder Sharma
16:50-17:15	AN-3023	Trace Identities in the Moduli Space of Kleinian Groups	Jianhua Gong
17:15-17:40	AN-3011	The $\Lambda$ -Aluthge Transformation of Closed Range Operators	Safa Menkad
17:40-18:05	AN-3021	Linear Functionals on Bochner-Lebesgue Spaces with Variable Exponent	Oscar Fonseca
<b>Session A2: Topology and Geometry</b> <b>Chair: Sadok Kallel</b> <b>Room: SBA0002</b>			
Time	ID	Title	Speaker
16:00-16:25	TG-3003	The Space of Two-Generator Kleinian Groups	Hala Alaqad
16:25-16:50	TG-3004	Some Operators and Limit Points of a Soft Set using Soft Somewhere Dense Sets	Aaesheh Zakari

16:50-17:15	TG-3005	On Fox-Trapezoidal Conjecture for Closed 3-Braids	Marwa Alrefai
17:15-17:40	TG-3006	Combinatorial Invariants of Stratified Spaces	Sadok Kallel
17:40-18:05	TG-3002	Rational Homotopy Methods in Graph Theory	Mahmoud Benkhalifa
<b>Special Session A2: Rings, Monoids and Module Theory</b> <b>Chair: Toma Albu</b> <b>Room: SBA0001</b>			
Time	ID	Title	Speaker
16:00-16:25	AA-3022	Torsion in Tensor Product and Rigid Ideals	Roger Wiegand
16:25-16:50	AA-3021	Vanishing of Tor over Fiber Products	Sylvia Wiegand
16:50-17:15	AA3082	Idealization of co-multiplication modules	Majid Ali
17:15-17:40	AA3031	Some finiteness conditions on the set of intermediate rings of a ring extension with zero divisors	Ali Jaballah
<b>Special Session A2: Partial Differential Equations, Analysis and Control</b> <b>Chair: Amjad Tuffaha</b> <b>Room: SBA0012</b>			
Time	ID	Title	Speaker
16:00-16:25	AM-3010	Some Aspects of Kuramoto-Sivashinsky Equations	Said Benachour
16:25-16:50	AM-3004	Evolution of Elementary Waves in Two Phase Mass Flows	Manoj Kumar Pandey
16:50-17:15	DE-3012	Analysis of Burgers- $\alpha$ Equation: Optimal Estimates of Parameter $\alpha$ using Physics-informed Deep Learning Algorithm	Bong-Sik Kim
17:15-17:40	AN-3019	Lower Semi-continuity and Convergence of a Class of Linear Growth Functionals with L1 Data	Thomas Wunderli
17:40-18:05	DE-3003	Global Existence and Stability for Coupled System of Hyperbolic Equations with Variable Exponents	Oulia Bouhoufani

<b>Special Session A2: Discrete Dynamic Modeling of Biological Systems</b> <b>Chair: Fil Castiglione</b> <b>Room: SBA0006</b>			
Time	ID	Title	Speaker
16:00-16:30	MB-3001	Network Control through Multistate Canalization	Elena Dimitrova
16:30-17:00	MB-3065	Network Reconstruction using Computational Algebra and Gene Knockouts	Matthew Macauley
17:00-17:30	MB-3002	Methylation Challenges & Opportunities for Biomarkers Identification – Focus on Imputation	Christine Nardini

17:30-18:00	MB-3006	Critical Nodes Reveal Remarkable Features of Human Essential Genes and Protein Interactome	Paolo Tieri
<b>Special Session A2: Advanced Numerical Methods and their Application</b> <b>Chair: Ali Sayfy</b> <b>Room: SBA0005</b>			
Time	ID	Title	Speaker
16:00-16:25	CM-3002	An Efficient approach for the solution of fractional BVPs	Ali Sayfy
16:25-16:50	CM-3006	Multilevel Iteration for the Nonlinear Mild-Slope Equation	Yogi A. Erlangga
16:50-17:15	CM-3001	Numerical Solution of Stochastic Partial Differential Systems with Additive Noise on Overlapping Subdomains	Mostafa Zahri
17:15-17:40	AM-3013	Numerical Determination of an Optimal Control for a Population Dynamics Model	M. Alahyane
17:40-18:05	CM-3004	Modeling and Dynamic Analysis of Two Weakly-Coupled Microbeams under Electrostatic Actuation	Muhannad Alkaddour
18:05-18:30	CM-3007	Dynamics of Metamaterial Beam Equipped with Vibration Absorbers	Ehab Basta

<b>Friday 7<sup>th</sup> February, 2020</b>			
<b>Registration</b> <b>8:00 – 09:00</b>			
<b>Session B1: Numerical Analysis</b> <b>Chair: Rama Bhargava</b> <b>Room: SBA0005</b>			
Time	ID	Title	Speaker

9:00-9:25	NA-3002	Instability Boundaries of Double-Diffusive Convection in a Brinkman Bidisperse Porous Medium with an anisotropic Permeability Effect	Sara Saleh
9:25-9:50	NA-3009	The Best Known Interior Point Algorithm for Linear Optimization Problem	El Amir Djaffal
9:50-10:15	NA-3008	A Discontinuous Galerkin Method for Systems of Stochastic Differential Equations with Applications to Population Biology, Finance, and Physics	Helmi Temimi
10:15-10:40	AM-3014	FEM Simulation on Nanofluid Flow over Power Law Stretching Sheet with MHD Thermo-Diffusive Effect	Rama Bhargava
<b>Session B1: Algebra and its Applications</b> <b>Chair: G.A. Pinto</b> <b>Room: SBA0011</b>			
Time	ID	Title	Speaker
9:00-9:25	AA-3007	Eventually Pointed Principally Ordered Regular Semigroups	G.A. Pinto
9:25-9:50	AA-3032	The Graded Module as a Clean Comodule	Nikken Puspita
9:50-10:15	AA-3033	On the Spectra of Mixed Extensions pf $P_3$	Sezer Sorgun
10:15-10:40	AA-3047	On Weakly Endocoprime Modules	Indah Wijayanti
<b>Special Session B1 : Rings, Monoids and Module Theory</b> <b>Chair: Ayman Badawi</b> <b>Room: SBA0001</b>			
Time	ID	Title	Speaker
9:00-9:25	AA-3013	A Gentle Introduction into CoGalois Theory	Toma Albu
9:25-9:50	AA-3051	Simultaneous Interpolation and p-adic Approximation by Integer-valued Polynomials	Sophie Frisch
9:50-10:15	AA-3012	Factorization behavior in Rings of Integer-valued Polynomials over Dedekind Domains.	Roswitha Rissner
10:15-10:40	AA-3002	On EM Conditions	Emad Abuosba

<b>Special B1: Partial Differential Equations, Analysis and Control</b> <b>Chair: Abdelaziz Soufyane</b> <b>Room: SBA0012</b>			
Time	ID	Title	Speaker
9:00-9:50	DE-3001	A Stability Result for a Nonlinear Damped Wave Equation with Variable-Exponent Nonlinearities	Salim Messoudi
9:50-10:15	AN-3017	Translation Operator and Maximal Function for the $(K, 1)$ -Generalized Fourier Transform	Salem Ben Said
10:15-10:40	FM-3001	Numerical Solution of an Integral Equation for Perpetual Bermudan Options	Ghada Alobaidi

## COFFEE BREAK

### 10:40-11:00 AM

#### Special Session B2: Rings, Monoids and Module Theory

Chair: Sophie Frisch  
Room: SBA0001

Time	ID	Title	Speaker
11:00-11:25	AA3058	Tilting Modules and Tilting Torsion Pairs	Alberto Tonolo
11:25-11:50	AA3039	Injective Modules over the Jacobson Algebra	Francesca Mantese
11:50-12:15	AA3056	Minimal Approximation of some Classes of Modules over Commutative Rings	Giovanna Le Gros

#### Special Session B2: Partial Differential Equations, Analysis and Control

Chair: Amjad Tuffaha  
Room: SBA0012

Time	ID	Title	Speaker
11:00-11:50	AM-3031	On some Inverse Boundary Value Problems related to the Monodomain Model of Cardiac Electrophysiology	Elena Beretta

## LUNCH BREAK

### 12:15-2:00 PM

#### Special Session B3: Rings, Monoids and Module Theory

Chair: Roger Wiegand  
Room: SBA0001

Time	ID	Title	Speaker
14:00-14:25	AA-3011	Cohen-Macaulay Unit Graphs of Commutative Rings.	T. Asir
14:25-14:50	AA-3060	Uniqueness of Zero-Divisor Graphs with Loops	Aihua Li
14:50-15:15	AA-3038	BZS Near-Rings and Rings	Mark Farag

#### Special Session B3: Partial Differential Equations, Analysis and Control

Chair: Ghada Alobaidi  
Room: SBA0012

Time	ID	Title	Speaker
14:00-14:50	DE-3022	Loss of Regularity for Transport Equations and Optimal Mixing	Anna Mazzucato
14:50-15:15	AN-3013	Some Questions related to Optimality of the Energy Behaviour of Reissner-Mindlin-Timoshenko Systems	Makram Hamouda
15:15-15:40	DE-3007	Global Attractors for Quasilinear Parabolic-Hyperbolic Equations Governing Longitudinal Motions of Nonlinearly Viscoelastic Rods	Suleyman Ulusoy

15:40-16:05	DE-3014	Eigenvalues of the Third Boundary Problem for Bitsadze Equation	Alip Mohamed
<b>Session B2: Applied Mathematics</b> <b>Chair: Rajinder Sharma</b> <b>Room: SBA0005</b>			
Time	ID	Title	Speaker
14:00-14:25	AM-3030	Flexural Vibration of Piezo Electric Solid Cylinder of Class 6 - Human Bone	Nehru Erode Santhanam
14:25-14:50	AM-3026	Existence and Approximate Controllability of Delayed Fractional Differential Equations with Deformable Derivatives	Dwijendra N. Pandey
14:50-15:15	AM-3016	Modeling HIV-TB Co-Infection with Illegal Immigrants	Rajinder Sharma
15:15-15:40	DE-3021	A Controllability Elucidation- Impulsive Fractional Higher Order Neutral Delay Differential Systems	B.Sundara Vadivoo
15:40-16:05	AM-3029	Fixed Point Theorem Applications in Combating Misinformation Spread through Social Media	Dubravka Gavric
<p style="text-align: center;"><b>Conference Dinner</b></p> <p style="text-align: center;"><b>Occidental Sharjah Grand Hotel</b></p>			

<b>Saturday 8<sup>th</sup> February, 2020</b>
<b>Registration</b> <b>8:00 – 09:00</b>
<b>Plenary Lecture II</b> <i>Permutation (Matrices) and Beyond</i> <b>Professor Richard Brualdi</b> <b>University of Wisconsin, Madison</b> <b>9:00-10:00</b>

Coffee Break & Group Photo Op 10:00 – 10:30			
Special Session C1: Designs, Codes and Graphs Chair: Maheshanand Bhaintwal Room: SBA0002			
Time	ID	Title	Speaker
10:30-10:55	AA-3049	Locally Recoverable Codes with Intersecting Recovering Sets	Maheshanand Bhaintwal
10:55-11:20	AA-3020	The Graphs Cospectral with the Pineapple Graph	Hatice Topcu
11:20-11:45	AA-3046	Vandermonde Sets and Hyperovals	Duy Ho
11:45-12:10	AA-3059	Hyperovals and Bent Functions	Kanat Abdukhalikov
12:10-12:35	AA-3043	$\mathbb{Z}_2\mathbb{Z}_4$ -Additive Quadratic Residue Codes	Taher Abualrub
Special Session C1: Rings, Monoids and Module Theory Chair: Marco Fontana Room: SBA0001			
Time	ID	Title	Speaker
10:30-10:55	AA-3050	Some Generalizations of Noetherian Rings	Jim Coykendall
10:55-11:20	AA-3018	Locally Free Cancellation for Definite Quaternion Algebras	Daniel Smertnig
11:20-11:45	AA-3035	Algebras whose Group of Units is Hyperbolic	Victor Bovdi
11:45-12:10	AA-3036	A Computation in Reflection Groups	Dong-il Lee
Session C1: Algebra and its Applications Chair: Zsolt Balogh Room: SBA0011			
Time	ID	Title	Speaker
10:30-10:55	AA-3040	Unitary Subgroups of Group Algebras	Zsolt Balogh
10:55-11:20	AA-3062	Classification of Covering Groups of Elementary Abelian 2-groups	Dana Saleh
11:20-11:45	AA-3048	An Introduction to Neutrosophic Semirings and Bisemirings	Inayatur Rehman

Session C1: Analysis Chair: Ilya Spitkovsky Room: SBA0010			
Time	ID	Title	Speaker
10:30-10:55	AN-3002	More Accurate Numerical Radius Inequalities (II)	Mohammad Sababheh
10:55-11:20	AN-3006	Estimation of Müntz Polynomials over the Intervals away from the Origin	Davit Martirosyan
11:20-11:45	AN-3004	On the Numerical Range of some Matrices with Scalar Diagonal Blocks	Ilya Spitkovsky

11:45-12:10	AN-3009	Chaotic P-Adic Dynamical Systems	Farrukh Mukhamedov
12:10-12:35	AN-3005	Analytic Families of Compact Operators Commuting with their Derivative	Abdelazi Maouche
<b>Session C1: Statistics</b> <b>Chair: Fadlalla Elfadaly</b> <b>Room: SBA0009</b>			
Time	ID	Title	Speaker
10:30-10:55	ST-3011	Frequency Polygon Estimator of the Mode of a Density Function under Weak Dependence	Ahmad Younso
10:55-11:20	ST-3024	On Efficiency of Split-Plot Response Surface Designs when some Observations are Missing	Yisa Yakubu
11:20-11:45	ST-3026	Locally Correct Confidence Intervals for a Binomial Proportion	Fadlalla Elfadaly
11:45-12:10	ST-3005	Skewed-Kotz Distribution with Application to Financial Stock Returns	Amadou Sarr
<b>Session C1: Numerical Analysis</b> <b>Chair: Vedat Suat Erturk</b> <b>Room: SBA0005</b>			
Time	ID	Title	Speaker
10:30-10:55	NA-3005	The Application of Differential Transform Method to a BVP arising in Chemical Reactor Theory	Vedat Suat Erturk
10:55-11:20	AM-3009	A Third-Order Shear Deformation Theory for Free Vibration Analysis of Functionally Graded Shells	Mohammad Zannon
11:20-11:45	CM-3003	Usage of the Randomized Kernel Functional Numerical Algorithm	V. Voytishek
11:45-12:10	NA-3003	A Family of Second Derivative Simpson's Type Block Methods for Stiff Systems	Yohanna Awari
12:10-12:35	NA-3007	Fourth Order Numerical Scheme for Two-Dimensional Inhomogeneous Distributed Order Riesz Space-Fractional Diffusion Equation	Muhammad Yousuf

<b>Session C1: Mathematical Biology</b> <b>Chair: Ziyad AlSharawi</b> <b>Room: SBA0006</b>			
Time	ID	Title	Speaker
10:30-10:55	MB-3013	Three Species Predator-Prey Interactions with Historic Behavior	Mansur Saburov
10:55-11:20	MB-3005	Edge Clustering Coefficient (ECC) based Node Centrality Measure to Identify Important Genes in Alzheimer's Disease	Ahmed Khasim
11:20-11:45	MB-3007	Mathematical Model of Glioma and Chemotherapy	Dua Alahmadi

11:45-12:10	MB-3011	Stability and Bifurcation Analysis of a Discrete-Time Predator-Prey Model with Strong Allee Effect	Ziyad AlSharawi
12:10-12:35	MB-3014	Lens Free High Resolution Computational Imaging using Fourier Techniques	Suhas P. Poyyilveetil
<b>Session C1: Mathematics Education</b> <b>Chair: Saadia Khouyibaba</b> <b>Room: SBA0007</b>			
Time	ID	Title	Speaker
10:30-10:55	ME-3004	How History of Mathematics Can Help in Teaching and Learning Mathematics	Saadia Khouyibaba
10:55-11:20	ME-3003	Concept-Based & Stem-Focused Math Teachers' Preparation Programs	Ali S. Shaqlaih
11:20-11:45	ME-3001	An Ethnomathematics Ios App Designed to Encourage Emirati Grade Six Students to continue taking Mathematics	Jason Johnson
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10:55-11:20	ST-3028	Fuzzy Time Series Forecasting Model Based on Singular Spectrum Analysis Decomposition	Subanar
11:20-11:45	AM-3024	Inference In Bayesian Networks: Junction Trees Constructions	Linda Smail
<b>Session C1: Partial Differential Equations, Analysis and Control</b> <b>Chair: Amjad Tuffaha</b> <b>Room: SBA0012</b>			
Time	ID	Title	Speaker
10:30-11:20	DE-3023	TBA	Nader Masmoudi
11:20-11:45	DE-3009	Memory-Type Boundary Control of a Laminated Timoshenko Beam	Abdelaziz Soufyane
11:45-12:10	AN-3017	A Convexity Problem for a Semi-Linear PDE	Layan Elhadj
<b>LUNCH BREAK</b> <b>12:30-2:00 PM</b>			
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14:00 – 14:25	AN-3026	On Logharmonic Mappings with Starlike Analytic Component	Zayid Abdelhadi
14:25 – 14:50	AN-3028	On the Commuting problem of Toeplitz operators acting on the Bergman space	Abdel Rahman Youssef

13:50 - 15:15	AN-3020	Geometric Studies of Normalized Modified Koebe Functions in Terms of Hypergeometric Functions	Firas Ghanim
15:15 - 15:40	AN-3025	Convolution of $\chi$ -orbital Measures on Complex Grassmannians	Mahmoud Al Hashami
15:40 - 16:05	AN-3027	Powers of Quasihomogeneous Toeplitz Operators	Issam Louhichi
<b>Session C2: Applied Mathematics</b> <b>Chair: Angel Tocino</b> <b>Room: SBA0005</b>			
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14:00-14:25	DE-3004	A Mathematical Approach for Turbulence and Enstrophy in the Taylor Green Vortex Model	Kamyar Mansour
14:25-14:50	AM-3017	Recent Developments on Fully Fuzzy Linear Programming	Mahmoud Alrefaei
14:50-15:15	AM-3020	On MS-Stability of Stochastic Differential Systems	Angel Tocino
15:15-15:40	AM-3025	Description of Ground State of $\lambda$ -Model on the Cayley Tree	Rauda Al Shamsi
15:40-16:05	AM-3028	Fractional Optimal Rearrangement Problems	Hayk Mikayelyan
<b>Session C2: Discrete Mathematics</b> <b>Chair: Mustapha Aouchiche</b> <b>Room: SBA0006</b>			
Time	ID	Title	Speaker
14:00-14:25	DM-3001	On the soEnergy of Graphs	Kahraman Birgin
14:25-14:50	DM-3002	The Edge Metric Dimension of some Subdivisions of the Wheel Graph	M.S. Bataineh
14:50-15:15	DM-3003	Nordhaus–Gaddum Inequality for the Spectral Radius of a Graph	Mustapha Aouchiche
15:15-15:40	DM-3004	On Partition Dimension of Infinite Graphs	Muhammad Imran
<b>Session C2: Probability Theory and Applications</b> <b>Chair: Guillaume Leduc</b> <b>Room: SBA0008</b>			
Time	ID	Title	Speaker
14:00-14:25	PA-3002	Hybrid Stochastic Differential Systems in Pharmacokinetics	Hana Baili
14:25-14:50	PA-3005	Derivative Formulae for Heat Semigroups on Riemannian Manifolds	James Thompson
14:50-15:15	PA-3006	Decidability of Learning in Finite Settings and Existence of Probabilities	Alberto Gandolfi
15:15-15:40	PA-3007	Convergence of lattice valued options to their Black-Scholes limit	Guillaume Leduc
<b>Session C2: Statistics</b> <b>Chair: Rafiq Hijazi</b> <b>Room: SBA0009</b>			
Time	ID	Title	Speaker

14:00-14:25	ST-3013	Assessment Practices in the Undergraduate Statistics Programs in the Arab World	Rafiq Hijazi
14:25-14:50	ST-3027	Modification of Generalized Space-Time Autoregressive Model for Prediction Monthly Incidence Rate in Banyumas Regency, Indonesia	Nunung Nurhayati
14:50-15:15	ST-3030	Multiple Regression Model to Examine the Incidence of Government Expenditure on Neonatal Mortality in Nigeria	Abubakar Usman
15:15-15:40	ST-3022	Zero-Inflated Models Application to Maternal Mortality Data	Kassim Tawiah
<b>Special Session C2: Rings, Monoids and Module Theory</b> <b>Chair: Jim Coykendall</b> <b>Room: SBA0001</b>			
Time	ID	Title	Speaker
14:00-14:25	AA-3042	Factorization of norms in rings of algebraic integers and weighted zero-sum problems	Wolfgang A. Schmid
14:25-14:50	AA-3028	On strongly primary monoids with a focus on puseux monoids	Felix Gotti
14:50-15:15	AA-3030	When is a Puiseux Monoid Atomic?	Marly Gotti
15:15-15:40	AA-3044	On the Notion of Krull Super-dimension	A.N. Zubkov
15:40-16:05	AA-3006	On 2-absorbing Ideals of Commutative Semiring	M. Saleh
<b>Special Session C2: Partial Differential Equations, Analysis and Control</b> <b>Chair: Cristian Enache</b> <b>Room: SBA0012</b>			
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14:00-14:25	DE-3005	Determining Functionals for a Non-Conservative, Nonlinear Plate Equation	Justin Webster
14:25-14:50	DE-3016	Global Well-Posedness of the Cauchy Problem for the Jordan-Moore-Gibson-Thompson Equation	Belkacem Said-Houari
14:50-15:15	AN-3010	An Introduction to Metrics and their Uses in Complex Analysis	Ziyad Adwan
15:15-15:40	DE-3015	A Monotonicity Property of the $\mathbb{P}$ -Torsional Rigidity	Cristian Enache

## Excursion: Global Village -Dubai

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## MODELLING COLLECTIVE CELL MIGRATION

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### **Abstract:**

Collective cell migration is a very common phenomenon in biology, occurring in embryology, wound healing and disease (cancer). This talk will review our recent work in two areas: (i) angiogenesis - the process by which new blood vessels are created by tumour cells. We will derive a new model for this phenomenon in the form of a coupled system of fully nonlinear partial differential equations. We will compare its behaviour with that of an existing model in the literature; (ii) cranial neural crest migration - we develop a simple agent-based model for this process and show how it can be used to generate new insights into the biology of this fundamental developmental process.

**Keywords:** *Collective Cell Migration, Nonlinear Partial Differential Equation*

**2010 Mathematics Subject Classification:** 93D15

**Biography:** Philip K. Maini received his Bachelor of Arts in Mathematics from Balliol College, Oxford, UK in 1982 and his DPhil in 1985 under the supervision of Professor J.D. Murray, FRS. In 1988 he was appointed Assistant Professor in the Mathematics Department at the University of Utah, Salt Lake City, USA. In 1990, he returned to Oxford as a University Lecturer and, in 1998, was appointed Professor of Mathematical Biology by Recognition of Distinction and Director of the Wolfson Centre for Mathematical Biology. In 2005, he was appointed Statutory Professor of Mathematical Biology. He is on the editorial boards of a large number of journals, including serving as Editor-in-Chief of the *Bulletin of Mathematical Biology* (2002–15). He is a SIAM Fellow, Fellow of the Royal Society (FRS), Fellow of the Academy of Medical Sciences (FMedSci), and Foreign Fellow of the Indian National Science Academy (FNA).

His present research projects include the modelling of avascular and vascular tumors, normal and abnormal wound healing, and a number of applications of mathematical modelling in pattern formation in early development, as well as the theoretical analysis of the mathematical models that arise in all these applications. He co-authored with Jonathan Sherratt and Paul Dale a Bellman Prize-winning paper (1997), was awarded a Royal Society Leverhulme Trust Senior Research Fellowship for 2001-2 and a Royal Society-Wolfson Research Merit Award (2006–11). In 2009, he was awarded the LMS Naylor Prize and Lectureship and in 2014 he was listed in "The World's Most Influential Scientific Minds 2014" (Thomson Reuters). In 2017, he was awarded the Arthur T. Winfree Prize from the Society of Mathematical Biology (SMB).

## PERMUTATION (MATRICES) AND BEYOND

**RICHARD A. BRUALDI**

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### **Abstract:**

Permutations are one of the most basic concepts in mathematics. Their study is both ancient and modern. They can be viewed as the integers  $1, 2, \dots, n$  in some order or as  $n \times n$  permutation matrices. They can be regarded as data which is to be sorted. The explicit definition of the classical determinant uses permutations. Associated with a permutation is the notion of an inversion (or interchange) where a larger integer precedes a smaller integer. Inversions can be used to define two partial orders on permutations, one weaker than the other. Partial orders have a unique minimal completion to a lattice, the Dedekind-MacNeille completion. Generalizations of permutation matrices determine related matrix classes. One of the above-mentioned partial orders gives rise to the generalization called alternating sign matrices (ASMs) that arose independently in the mathematics and physics literature. Permutations may contain certain patterns, e.g. three integers in increasing order; avoiding such patterns determines certain permutation classes. Similar restrictions can be placed more generally on  $(0,1)$ -matrices. The convex hull of  $n \times n$  permutation matrices is the polytope of  $n \times n$  doubly stochastic matrices. In a similar way we get ASM polytopes. The purpose of this lecture is to explore these and other ideas and their connections.

**Keywords:** Alternating Sign Matrices, Inversion.

**2010 Mathematics Subject Classification:** 15A09

**Biography:** Richard A. Brualdi is UWF Beckwith Bascom Professor of Mathematics 2004-2008 (now emeritus) at the University of Wisconsin-Madison (USA). He received the Chancellor's Award for Excellence in Teaching at UW-Madison in 1986, the Euler Medal in 2000 for a lifetime career of distinguished contributions to combinatorial research by a member of The Institute of Combinatorics and its Applications (ICA), and the International Linear Algebra (ILAS) Prize in 2005 for lifetime contributions. He is a fellow of the Society for Industrial and Applied Mathematics (SIAM) and Fellow of the American Mathematical Society (AMS). He is a co-editor-in-Chief of the journal "Linear Algebra and Its Applications" and a co-editor-in-chief of the "Electronic Journal of Combinatorics". He is the author of over 200 papers and 6 books. He has had 37 PhD students at the University of Wisconsin-Madison.

# Algebra and its Applications

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9:00-9:25	AA-3007	Eventually Pointed Principally Ordered Regular Semigroups	G.A. Pinto
9:25-9:50	AA-3032	The Graded Module as a Clean Comodule	Nikken Puspita
9:50-10:15	AA-3033	On the Spectra of Mixed Extensions of $P_3$	Sezer Sorgun
10:15-10:40	AA-3047	On Weakly Endocoprime Modules	Indah Wijayanti

<b>Session C1: Algebra and its Applications</b> <b>Chair: Zsolt Balogh</b> <b>Room: SBA0011</b>			
<b>Time</b>	<b>ID</b>	<b>Title</b>	<b>Speaker</b>
10:30-10:55	AA-3040	Unitary Subgroups of Group Algebras	Zsolt Balogh
10:55-11:20	AA-3062	Classification of Covering Groups of Elementary Abelian 2-groups	Dana Saleh
11:20-11:45	AA-3048	An Introduction to Neutrosophic Semirings and Bisemirings	Inayatullah Rehman

## EVENTUALLY POINTED PRINCIPALLY ORDERED REGULAR SEMIGROUPS

**G. A. Pinto**

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Sultanate of OMAN

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☒ X Oral presentation

☐ Poster Presentation

### **Abstract:**

An ordered regular semigroup,  $S$ , is said to be *principally ordered* if, for every  $x \in S$  there exists  $x^* = \max\{y \in S \mid xyx \leq x\}$ . A principally ordered regular semigroup is *pointed* if for every element  $x$ , we have  $x^2 \leq x$ . In this talk we investigate those principally ordered regular semigroups that are *eventually pointed* in the sense that for all  $x \in S$  there exists a positive integer,  $n$ , such that  $(x^n)^2 \leq x^n$ .

**Keywords:** Ordered Regular Semigroups

**2010 Mathematics Subject Classification:**

### **References:**

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## THE GRADED MODULE AS A CLEAN COMODULE

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### Abstract:

Let  $C$  be a coassociative and counital  $R$ -coalgebra which satisfies the  $\alpha$ -condition. A clean module is defined based on the notion of clean rings [1, 2, 3, 4, 5, 6]. An  $R$ -module  $M$  is a clean module if the ring of endomorphism of  $M$  over  $R$  is clean [7]. Furthermore, since any  $C$ -comodule  $M$  can be considered as a module over the dual algebra  $C^* = \text{Hom}_R(C, R)$ , we introduced clean comodules as follow. A clean (right)  $C$ -comodule  $M$  is called a clean  $C$ -comodule provided  $M$  is clean as a (left) module over the dual algebra  $C^*$ . Suppose that  $G$  is a semigroup and consider  $R$  as a  $G$ -graded ring by trivial grading. We are already known that any  $G$ -graded module over  $R$  is an  $R[G]$ -comodule, and it is true for the converse [8]. We give a sufficient condition of clean comodules over coalgebra of semigroup ring  $R[G]$ . By observed some properties in category theory and category of graded modules [9,10], we showed that every  $G$ -graded module  $M$  over  $R$  is a clean comodule over semigroup ring  $R[G]$  if  $M$  is a clean  $R$ -module.

**Keywords:** clean comodule, clean module, semigroup ring.

**2010 Mathematics Subject Classification:** 16T15, 16D90, 16W50

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## ON THE SPECTRA OF MIXED EXTENSIONS OF $P_3$

Sezer Sorgun

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### Abstract:

A mixed extension of a graph  $G$  is a graph  $H$  obtained from  $G$  by replacing each vertex of  $G$  by a clique or a coclique, whilst two vertices in  $H$  corresponding to distinct vertices  $x$  and  $y$  of  $G$  are adjacent whenever  $x$  and  $y$  are adjacent in  $G$ . Using the classification given by Haemers [5] we investigate mixed extension of  $P_3$  on being determined by the adjacency spectrum. We present several cospectral families, and with the help of a computer we find all graphs on at most 25 vertices that are cospectral with a mixed extension of  $P_3$ .

**Keywords:** Graph, Mixed extension, Graph spectrum.

**2010 Mathematics Subject Classification:**05C50

### References

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ON WEAKLY ENDOCOPRIME MODULES

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**Abstract:**

Let  $M$  be a right  $R$ -module and  $S$  be an endomorphism ring of  $M_R$ . In this paper, a dualization of weakly endoprime modules is given. A module  $M$  is called a weakly endocoprime module if for every proper fully invariant submodule  $N$  of  $M$ , the annihilator of module factor  $M/N$  of  $S$  is a prime ideal. Moreover, we give some properties of weakly endocoprime modules and define a weakly endocoprime radical of  $M$  as the sum of all weakly endocoprime submodule of  $M$ . We present some properties of weakly endocoprime radicals of a module and we give a necessary condition for a module being weakly endocoprime radical module.

**Keywords:** weakly endocoprime; weakly endoprime; weakly endocoprime radical.

**2010 Mathematics Subject Classification:** 16D10, 16D90, 16N60

**References**

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## UNITARY SUBGROUPS OF GROUP ALGEBRAS

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**Abstract:** Let  $FG$  be a group algebra of a finite  $p$ -group  $G$  over the field  $F$  of  $p$  elements. The algebra  $FG$  has the classical involution  $*$ , which is a linear extension of a group involution acting on  $G$ , sending each element to its inverse. Denote by  $V(FG)$  the group of normalized units in  $FG$ . The  $*$ -unitary subgroup of  $FG$ , denoted by  $V_*(FG)$ , is defined to be the set of all normalized units  $u$  satisfying the property  $u^* = u^{-1}$ . In this talk we intent to deal with the structure of unitary subgroup of group algebras. We also give a recursive method how to compute the order of the  $*$ -unitary subgroup for some non-commutative group algebras.

**Keywords:** Group Algebra, Involutions, Unitary subgroup

**2010 Mathematics Subject Classification:** 16S34, 16U60.

## Classification of covering groups of elementary abelian 2-groups

Dana Saleh, Rachel Quinlan

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**Abstract:** A covering group (or Schur cover) of the elementary abelian  $p$ -group of order  $p^n$  is a group  $G$  with the following properties:

1.  $G$  is generated by  $n$  elements  $x_1, \dots, x_n$ .
2. The centre of  $G$  is equal to the commutator subgroup  $G'$  and is elementary abelian of order  $p^{n(n-1)/2}$ , generated by the  $n(n-1)/2$  simple commutators  $[x_i, x_j]$ .
3.  $G/G'$  is elementary abelian of order  $p^n$ .

In general, an elementary abelian group has many non-isomorphic covering groups whose enumeration and/or classification is a difficult problem. Different covering groups are determined by specifying the  $p$ th powers of the generators  $x_i$  as elements of the elementary abelian group  $G'$ . In odd characteristic, the problem can be expressed purely in terms of linear algebra, because the mapping from  $G$  to  $G'$  that takes every element to its  $p$ th power is a linear transformation of  $F_p$ -vector spaces. In characteristic 2, this is not the case and the data has a more combinatorial flavour. An invariant of covering groups of  $C_2^n$  is the minimum number  $k$  of elements with distinct squares in a generating set. In the case  $k = 1$ , the corresponding covering groups are called uniform and it is known that their isomorphism types are in bijective correspondence with the isomorphism types of simple undirected graphs on  $n$  vertices [1]. This talk will report some recent progress on classifying groups with graphs in the almost uniform case  $k = 2$ .

**Keywords:** elementary abelian group, Schur covering group.

**2010 Mathematics Subject Classification:** 20K01, 05C25

## References

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## AN INTRODUCTION TO NEUTROSOPHIC SEMIRINGS AND BISEMIRINGS

<sup>1</sup>Inayatullah Rehman, <sup>2</sup>Muhammad Asif Gondal, <sup>3</sup>Muhammad Gulistan and <sup>4</sup>Asima Razzaque

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### Abstract:

In this paper we initiated the concept of Neutrosophic Semirings (SUI,  $\ast_1$ ,  $\ast_2$ ) and Neutrosophic Bisemirings (SUI,  $\ast_1$ ,  $\ast_2$ ,  $\ast_3$ ). The substructures of each structure have been defined and some useful results have been proved. Moreover, in order to familiarize the readers with these concepts some worthy examples have been provided. The left, right and two sided ideals of Neutrosophic Semirings and Neutrosophic Bisemirings have been paid a special attention. Finally, we turned our discussion towards the compatible and congruence relations and intuitively some remarkable properties have also been considered. We have provided many examples to express the rationality of each notion discussed in this paper.

**Keywords:** Neutrosophic semirings, neutrosophic bisemirings, ideals and congruence relations

**2010 Mathematics Subject Classification:** 12K10

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# Analysis

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<b>Session A2: Analysis</b> <b>Chair: Oscar Fonseca</b> <b>Room:SBA0010</b>			
Time	ID	Title	Speaker
16:00-16:25	AN-3014	Approximation Properties of $\Theta$ -Means of Walsh-Fourier Series in Different Spaces	Karoly Nagy
16:25-16:50	AN-3015	Some Fixed Point Results in Complex Valued Dislocated Metric Space and its Applications	Rajinder Sharma
16:50-17:15	AN-3023	Trace Identities in the Moduli Space of Kleinian Groups	Jianhua Gong
17:15-17:40	AN-3011	The $\Lambda$ -Aluthge Transformation of Closed Range Operators	Safa Menkad
17:40-18:05	AN-3021	Linear Functionals on Bochner-Lebesgue Spaces with Variable Exponent	Oscar Fonseca
<b>Session C1: Analysis</b> <b>Chair: Ilya Spitkovsky</b> <b>Room:SBA0010</b>			
Time	ID	Title	Speaker
10:30-10:55	AN-3002	More Accurate Numerical Radius Inequalities (II)	Mohammad Sababheh
10:55-11:20	AN-3006	Estimation of Müntz Polynomials over the Intervals away from the Origin	Davit Martirosyan
11:20-11:45	AN-3004	On the Numerical Range of some Matrices with Scalar Diagonal Blocks	Ilya Spitkovsky
11:45-12:10	AN-3009	Chaotic P-Adic Dynamical Systems	Farrukh Mukhamedov
12:10-12:35	AN-3005	Analytic Families of Compact Operators Commuting with their Derivative	Abdelazi Maouche
<b>Session C2: Analysis</b> <b>Chair: Zayid Abdelhadi</b> <b>Room:SBA0010</b>			
Time	ID	Title	Speaker
14:00 – 14:25	AN-3026	On Logharmonic Mappings with Starlike Analytic Component	Zayid Abdelhadi
14:25 – 14:50	AN-3028	On the Commuting problem of Toeplitz operators acting on the Bergman space	Abdel Rahman Youssef
13:50 - 15:15	AN-3020	Geometric Studies of Normalized Modified Koebe Functions in Terms of Hypergeometric Functions	Firas Ghanim
15:15 - 15:40	AN-3025	Convolution of $\chi$ -orbital Measures on Complex Grassmannians	Mahmoud Al Hashami
15:40 - 16:05	AN-3027	Powers of Quasihomogeneous Toeplitz Operators	Issam Louhichi

## Approximation properties of $\Theta$ -means of Walsh-Fourier series in different spaces

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**General area of research:** Analysis

**Abstract:** We discuss the behavior of  $\Theta$ -means of Walsh-Fourier series of a function in  $L^p$  spaces [1]. This result was improved to (dyadic) homogeneous Banach spaces [2] and dyadic Hardy spaces. Namely, we estimate the rate of the approximation of  $\Theta$ -means in terms of modulus of continuity.

It is a generalization of results of Móricz, Siddiqi [5], Fridli, Manchanda, Siddiqi [3] on Nörlund means and Móricz, Rhoades on weighted means [4].

**Keywords:** homogeneous Banach spaces, dyadic Hardy spaces,  $\Theta$ -mean

**2010 Mathematics Subject Classification:** 42C10

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## SOME FIXED POINT RESULTS IN COMPLEX VALUED DISLOCATED METRIC SPACE AND ITS APPLICATIONS

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### **Abstract:**

In this paper, we established some common fixed point theorems for a pair of self mappings in a complex dislocated metric space. We generalize the results proven in [1] to a pair of self maps. An applications of the proven results to differential equations and iterated functions is also provided.

**Keywords:** Common fixed point, Dislocated metric space, Banach contraction principle.

**2010 Mathematics Subject Classification:** 47H10, 54H25

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## TRACE IDENTITIES IN THE MODULI SPACE OF KLEINIAN GROUPS

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**Abstract:** We shall discuss in this talk a very interesting family of polynomial trace identities which will be used to obtain geometric information about Kleinian groups. This is the joint work with Hala Alaquad and Gaven Martin, and this project is supported by UAE University research grant UPAR G00002670.

**Keywords:** Trace identity, moduli space, Kleinian group.

**2010 Mathematics Subject Classification:** 30F40, 22E40

## The $\lambda$ -Aluthge transformation of colsed range operators

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**Abstract:** Let  $T = U|T|$  be the polar decomposition of a bounded linear operator  $T$  on a Hilbert space  $H$ . Then, for every  $\lambda \in [0, 1]$  the  $\lambda$ -Aluthge transformation of  $T$  is defined by  $\Delta_\lambda(T) = |T|^\lambda U |T|^{1-\lambda}$ . This notation was first introduced by Aluthge in the case when  $\lambda = \frac{1}{2}$  in [1] and it is a powerful tool in operator theory. An operator  $T$  is said to be binormal if  $|T||T^*| = |T^*||T|$ . In this paper, we study the class of binormal bounded linear operators with closed range via  $\lambda$ -Aluthge transformation and generalised inverse.

**Keywords:** Binormal operator,  $\lambda$ -Aluthge transformation, Moore-Penrose inverse.

**2010 Mathematics Subject Classification:** Primary 47B33; Secondary 47B38

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## LINEAR FUNCTIONALS ON BOCHNER-LEBESGUE SPACES WITH VARIABLE EXPONENT

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□ Oral presentation

### Abstract:

In this talk we introduce the space of bounded variation vector measures in the Riesz sense with variable exponent ( $RBV_{p(\cdot)}$ ). A characterization of the linear functionals on Bochner-Lebesgue spaces with variable exponent is given in terms of ( $RBV_{p(\cdot)}$ ).

**Keywords:** :Riesz bounded variation, vector measures, variable exponent Bochner-Lebesgue space.

**2010 Mathematics Subject Classification:** 46E30.

### References

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## More accurate numerical radius inequalities (II)

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### Abstract:

In a recent work of the authors, we showed some general inequalities governing numerical radius inequalities using convex functions. In this article, we present results that complement the aforementioned inequalities. In particular, the new versions can be looked at as refined and generalized forms of some well known numerical radius inequalities. Among many other results, we show that

$$\left\| f\left(\frac{A^*A + AA^*}{4}\right) \right\| \leq \left\| \int_0^1 f\left((1-t)B^2 + tC^2\right) dt \right\| \leq f\left(w^2(A)\right),$$

when  $A$  is a bounded linear operator on a Hilbert space having the Cartesian decomposition  $A = B + iC$ . This result, for example, extends and refines a celebrated result by Kittaneh.

**Keywords:** Numerical radius, Hermite-Hadamard inequality, operator convexity.

**2010 Mathematics Subject Classification:** 47A12, 47A30, 15A60, 47A63.

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## ESTIMATION OF MÜNTZ POLYNOMIALS OVER THE INTERVALS AWAY FROM THE ORIGIN

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In Hilbert spaces some special biorthogonal systems play the same important role as orthogonal systems usually do when question concerns to extremal problems. Let  $\mu_1, \mu_2, \dots, \mu_n$  be distinct complex numbers with real parts greater than  $-0.5$ . Then for the finite Müntz system

$$\{x^{\mu_k}\}_{k=1}^n \quad (1)$$

one can construct the unique system  $\{\varphi_k\}_{k=1}^n$  which is biorthogonal to (1) in Lebesgue space  $L^2(0, 1)$  and belongs to the linear span of (1). The system  $\{\varphi_k\}_{k=1}^n$  has a comfortable integral representation (see [1]). When the biorthogonal system is asked to be found in  $L^2(\eta, 1)$  for  $\eta \in (0, 1)$ , the method developed in [1] is no longer applicable (see [2]).

Our research is currently focused on biorthogonalization of system (1) over the intervals away from the origin. A workable form for the biorthogonal system generated by (1) is sought to estimate Müntz polynomials. In this sense, the following new result serves as a departure point.

**Theorem 1** *Let  $\mathcal{M}$  be the set of Müntz polynomials generated by (1) and  $G_\eta$  be the Gram matrix of (1) in  $L^2(\eta, 1)$ ,  $\eta \in (0, 1)$ . Then for any  $x \in (0, \infty)$  the equality*

$$\max_{P \in \mathcal{M}} \frac{|P(x)|}{\|P\|_{L^2(\eta, 1)}} = \sqrt{\Phi_\eta(x)} \quad (2)$$

holds, where

$$\Phi_\eta(x) = -\frac{1}{\det(G_\eta)} \cdot \det \begin{bmatrix} 0 & x^{\bar{\mu}_1} \dots x^{\bar{\mu}_n} \\ x^{\mu_1} & \\ \vdots & \\ x^{\mu_n} & \end{bmatrix} G_\eta.$$

Further questions are explored such as extending the theorem for complex  $x$  over the Riemann surface of logarithmic function and finding for each  $x$  the extremal Müntz polynomial where the maximum in (2) is attained. The launched method can be used to establish analogs of Markov-Bernstein and Chebyshev-type inequalities reviewed in [3].

**Keywords:** Müntz Polynomials, Biorthogonal Systems, Gram Matrix.

**2010 Mathematics Subject Classification:** 30B50, 41A17, 42C05

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# ON THE NUMERICAL RANGE OF SOME MATRICES WITH SCALAR DIAGONAL BLOCKS

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## Abstract:

Several new verifiable conditions are established for matrices of the form  $\begin{bmatrix} \alpha I_{n-k} & C \\ D & \beta I_k \end{bmatrix}$  to have the numerical range equal the convex hull of at most  $k$  ellipses. For  $k = 2$ , these conditions are also necessary, provided that the ellipses are co-centered. The results obtained extend and unify those obtained in [1]–[6].

**Keywords:** Numerical range, Schur complement, Ellipticity

**2010 Mathematics Subject Classification:** 15A60

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## CHAOTIC P-ADIC DYNAMICAL SYSTEMS

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### **Abstract:**

There are many investigations that have been conducted to discuss and debate the question due to the assumption that  $p$ -adic numbers provide a more exact and more adequate description of microworld phenomena [1,2]. Consequently, various models in physics described in the language of  $p$ -adic analysis (see e.g. [3,4]) which propose to investigate  $p$ -adic dynamical systems. Recently, polynomials and rational maps of  $p$ -adic numbers have been studied as dynamical systems over this field [5]. It turns out that these  $p$ -adic dynamical systems are quite different to the dynamical systems in Euclidean spaces. In theoretical physics, the interest in  $p$ -adic dynamical systems was started with the development of  $p$ -adic models [6]. In these investigations, the importance of detecting chaos was stressed in the  $p$ -adic setting [5,6]. In this talk we are interested in chaotic behavior of certain  $p$ -adic dynamical systems associated with  $p$ -adic lattice models.

**Keywords:** chaos;  $p$ -adic numbers; dynamical systems;

### **2010 Mathematics Subject Classification:**

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## ANALYTIC FAMILIES OF COMPACT OPERATORS COMMUTING WITH THEIR DERIVATIVE

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### **Abstract:**

Spectral properties of analytic families of compact operators on a Hilbert space are studied. The results obtained are then used to establish that an analytic family of self-adjoint compact operators on a Hilbert space  $H$ ; which commute with their derivative, must be functionally commutative.

In [1], Stuart Goff studied analytic hermitian function matrices which commute with their derivative on some real interval.

He obtained as a main result that these matrices are functionally commutative on  $I$ ; i.e.,

$A(s)A(t) = A(t)A(s)$  for all  $s, t$  in  $I$  [[2], Theorem 3.6].

Subsequently, in [1], Jean-Claude replaced the interval  $I$  by an open connected subset of a Banach space on  $\mathbb{R}$  or  $\mathbb{C}$  and generalized Goff's theorem in ([1], Theorem 4.3).

He also summarizes the history and motivations behind the problem on matrix functions commuting with their derivative from 1950 to 1982.

Our aim is to further extend the result of Goff from matrices to the infinite-dimensional situation of compact self-adjoint operators on a separable Hilbert space.

We study first analytic families of compact self-adjoint operators on a complex Hilbert space, which commute with their derivative on some real interval  $I$ :

Our main result establishes that these operators must be functionally commutative on  $I$ ; that is,  $A(s)A(t) = A(t)A(s)$  for all  $s$  and  $t$  in  $I$ .

This of course extends the main result of [2] and [1] from the case of matrices to the infinite dimensional situation of operators on a Hilbert space.

Then, we will explain how to solve the general problem when we consider only self-adjoint operators on a separable Hilbert space (without the compactness hypothesis), and comment on the other more general extension of our result to Analytic families of compact operators on a Banach space.

**Keywords:** Compact operator, analytic multivalued function, projection.

**2010 Mathematics Subject Classification:**

### **References:**

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On logharmonic mappings with starlike analytic component

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**Abstract:** In this paper we consider the set  $S_L$  of all logharmonic mappings defined on the unit disk  $U$  which are of the form  $f(z) = h(z)\overline{h'(z)}$ , where  $h(z)$  is starlike analytic mapping. A detail study of the class  $S_L$  will be given. In particular, the radius of starlikeness for the class  $S_L$  is determined. Moreover, a sharp estimate for the arclength for functions in this class are established. Additionally, Bohr's theorem for the class  $S_L$  will be investigated.

**Keywords:** logharmonic mappings, univalent, starlike, Bohr's Theorem.

**2010 Mathematics Subject Classification:** 30C35, 30C45

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## On the Commuting problem of Toeplitz operators acting on the Bergman space

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**General area of research:** Analysis

### Abstract:

Various algebraic problems related to Toeplitz operators have been extensively studied in the literature. One of the most interesting problems in the field is the commuting problem. Recall that two operators  $A$  and  $B$  commute if and only if their commutator  $[A, B] = AB - BA = 0$ . This problem was motivated by the same problem for Toeplitz operators on the Hardy space over the unit circle, which was completely solved by Brown and Halmos in [3]. This problem is still wide open for Toeplitz operators acting on the Bergman space.

In this talk, we present recent results and contributions obtained so far toward solving the commuting problem of Toeplitz operators on the Bergman space.

**Keywords:** Toeplitz operator, Bergman Space, Commuting problem.

**2010 Mathematics Subject Classification:**

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## GEOMETRIC STUDIES OF NORMALIZED MODIFIED KOEBE FUNCTIONS IN TERMS OF HYPERGEOMETRIC FUNCTIONS

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□ Oral presentation

### Abstract:

The current work, corresponding to convolution gadget and hypergeometric functions, introduces a new normalized modified Koebe function in the complex open unit disk. Furthermore, the stipulations on parametrers of the modified Koebe function to be star-like, convex and close-to-convex are discussed and examined. The stipulations on modified Koebe function to be include in the Hardy space are also acquired.

**Keywords:** Koebe Functions, hypergeometric Functions, convolution.

**2010 Mathematics Subject Classification:** 30C45, 30C50, 30C10

### References

[1]

[2]

[3]

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## Convolution of $\chi$ -orbital Measures on Complex Grassmannians

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**Abstract:** The regularity of the Radon-Nikodym derivative of a convolution of orbital measures on compact symmetric spaces  $U/K$  was considered by many authors. In an earlier work we considered the mentioned problem on complex Grassmannians. Recently, we extended our work to the context of  $\chi$ -orbital measures. More precisely, for  $U$  be a compact Lie group,  $K$  a compact subgroup of  $U$ , and  $\chi : K \rightarrow \mathbb{C}$  a character, we introduced a new class of orbital measures, which we called " $\chi$ -orbital measures", on  $U$  and we studied the regularity of the Radon-Nykodim derivative of a convolution of such measures with respect to the Haar measure of  $U$ . The focus was on complex Grassmannians.

The aim of the talk is to explain the main ideas of our work.

**Keywords:**  $\chi$ -Orbital measures, Radon-Nikodym derivative, Complex Grassmannians.

**2010 Mathematics Subject Classification:** Primary 43A77, 43A90; Secondary 53C35, 28C10

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## POWERS OF QUASIHOMOGENEOUS TOEPLITZ OPERATORS

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### **Abstract:**

In this talk, we shall present sufficient conditions for the existence of powers of quasihomogeneous Toeplitz operators defined on the Bergman space of the unit disk of the complex plane. A large class of examples shall be provided to illustrate our results. To our best knowledge those examples are not covered by the current literature. This is a joint work with Aissa Bouhali and Zohra Bendaoud from Algeria.

**Keywords:** Toeplitz Operators.

# Applied Mathematics

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## HYDROMAGNETIC AND THERMAL BOUNDARY LAYER FLOW DUE TO RADIAL STRETCHING SHEET WITH DUFOUR AND SORET EFFECTS

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### **Abstract:**

In this paper, the problem of Hydromagnetic and Thermal Boundary Layer Flow Due to Radial Stretching Sheet with Dufour and Soret Effects was analyzed using the Adomian Decomposition. The governing partial differential equations (PDEs) were reduced with the help of similarity variables to non linear coupled ordinary differential equations (ODEs). The influences of various physical parameters were presented numerically and graphically. Numerical comparisons were carried out with the existing literature and a good agreement was established. The magnetic parameter was found to be a reduction agent of the velocity profile.

**Keywords:** Radial stretching, Stagnation point, Hydromagnetic.

**2010 Mathematics Subject Classification:** 76-XX

## AUTOIONIZATION OF HOLLOW ATOMIC IONS

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### Abstract:

A time-dependent close-coupling method is developed to calculate total, double and triple autoionization rates for hollow atomic ions of four-electron systems. This work was motivated by recent observations of four-electron Auger process in near K-edge photoionization of  $C^+$  ions [1]. The time-dependent close-coupled equations are solved using lattice techniques to obtain discrete representation of radial wave functions and all operators on four dimensional grid with uniform spacing. Initial excited states are obtained by relaxation of the Schrödinger equation in imaginary time using a Schmidt orthogonalization method involving interior subshells. The radial wave function grids are partitioned over the cores on a massively parallel computer, which is essential due to the large memory requirements needed to store the coupled wave functions and the long run times needed to reach convergence of the ionization process. Total, double, and triple autoionization rates are obtained by propagation of the time-dependent close-coupled equations in real time using integration over bound and continuum single particle states. These states are generated by matrix diagonalization of one-electron Hamiltonians. The total autoionization rates for each  $\mathcal{L}$  excited state is found to be slightly above the single autoionization rate for the excited configuration using configuration-average distorted-wave theory. As expected, we find the double and triple autoionization rates to be much smaller than the total autoionization rates [2]. Future work can be extended to study electron-impact triple ionization of atoms or ions.

The work was supported in part by grants from American University of Sharjah and the US Department of Energy. Computational work was carried out at the National Energy Research Scientific Computing Center (NERSC) in Berkeley, California, USA.

**Keywords:** Hollow atoms, autoionization, Auger rates.

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## CFD – A TOOL TO STUDY AIR POLLUTION PROBLEMS

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### **Abstract:**

Fluid dynamics is the science of fluid motion. Fluid (gas and liquid) flow is commonly studied in one of three ways:

- Theoretical
- Experimental
- Numerical: Computational Fluid Dynamics (CFD).

The complexity of the governing equations in theoretical development generally allows analytical solutions to be obtained only for very simple cases and experimentation is usually too involved/expensive as well as the growing need to optimize systems has made it imperative to numerically simulate the fluid flow. Thus, emerged and yet emerging discipline called CFD.

CFD is separate from each theoretical and experimental branches of Fluid Dynamics, although it has aspects of both, and that it supplements rather than replaces, offering new perspectives in the study of physical processes.

Fluid flows are governed by partial differential equations (PDE) which represent conservation laws for the mass, momentum, and energy. CFD is, in part, the art of replacing such PDE systems by a set of algebraic equations which can be solved using digital computers. CFD are intimately related to advances in computer hardware, particularly in regard to storage and execution speed. Such solutions are only approximate solution of an exact equation. However, the key point is that, we can often make an assessment of the magnitude of errors and can always improve the accuracy.

CFD provides a qualitative and quantitative prediction of fluid flows by means of:

- mathematical modeling (partial differential equations)
- numerical methods (discretization and solution techniques)
- software tools (solvers, pre and post-processing utilities)

CFD is a highly interdisciplinary research area which lies at the interface of physics, applied mathematics, and computer science.

**This paper discuss the use of CFD, to study air pollution problems in an urban area,** which in turn provides an acceptable indication of distribution of atmospheric pollutants much more easily and quickly than monitoring networks.

**Keywords:** CFD;Mathematical Models;Air Pollution

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2. *Numerical Heat Transfer and Fluid Flow*; Computational Methods in Mechanics and Thermal Science; S.V. Patankar; Hemisphere Pub, 1980

## STOCHASTIC DELAY DIFFERENTIAL EQUATIONS FOR THREE SPECIES PREY-PREDATOR SYSTEM WITH ALLEE EFFECT

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### Abstract:

Stochastic delay differential equations (SDDEs) can provide an additional degree of realism compared to their corresponding deterministic counterpart because of the randomness and stochasticity of real life. In this work, we study the dynamics of a stochastic delay differential model for two prey-one predators where the growth of both preys' populations is subjected to Allee effect. We show that there is a unique global positive solution of the system, with positive initial conditions. Sufficient conditions for stochastically ultimately bounded in mean and almost surely asymptotic properties, using Lyapunov functional, are obtained. The increase of the noise intensity has a drastic impact on the dynamical behavior of species with or without the delay effect. Time-delay plays a vital role in population dynamics of prey-predator, which has been recognized to contribute critically to the stable or unstable outcomes of preys' population due to predation. Additionally, it has been seen that small noises can support the survival of species; While large noises can lead to extinction. Some illustrative numerical simulations, using Milstein's scheme, are carried out to show the effectiveness of the theoretical results.

**Keywords:** Allee effect; Milstein's scheme; Prey-predator system; Stochastic DDEs

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## FLEXURAL VIBRATION OF PIEZO ELCTRIC SOLID CYLINDER OF CLASS 6 - HUMAN BONE

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### **Abstract:**

This paper presents a Flexural vibration of Piezo Electric Solid Cylinder of Class -6-Human Bone. The frequency equations are obtained for the traction free surfaces with continuity condition at the interfaces. The boundary conditions are solved by using Fourier Collocation Method. The frequency equation is solved by using Muller's Method. In this paper we studied about the attenuation effect and Vibration characteristic for different wave numbers. Numerical results are carried out for the Human Bone material constants and the dispersion curves are compared with that of a solid piezoelectric cylinder and a similar model embedding a Corban Fiber Reinforced Plastic (CFRP).

**Keywords:** Flexural Vibration, Human bone, Piezoelectric

**2010 Mathematics Subject Classification:** 74H50

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## Existence and approximate controllability of delayed fractional differential equations with deformable derivatives

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**Abstract:** The main objective of this article is to provide the set of sufficient conditions for the existence of mild solution of Atangana-Baleanu fractional differential system with non-instantaneous impulses. Results are obtained via non-compactness of the semigroup and fixed point theory. In the end, an example is given to justify the theoretical results.

**Keywords:** Atangana-Baleanu derivative, fractional differential equations, fixed point theorems.

**2010 Mathematics Subject Classification:** 58C30, 34A08, 34K37.

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## MODELING HIV-TB CO INFECTION WITH ILLEGAL IMMIGRANTS

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### Abstract:

This study deals with the formulation of a non-linear mathematical model to understand the transmission dynamics and prevalence of HIV-AIDS and a curable TB pathogen infection in presence of illegal migrants. The incorporation of the factor, illegal immigrants staying illegally after the expiry of residence visa without performing HIV –AIDS and TB detection medical test makes our model more closer to the real life situations. Using stability theory ,the analysis of the model has been done by finding out all the equilibrium points of the system. The stability analysis has also been done for the parameters involved in the model. We discuss the role of Illegal Immigrants staying illegally without performing HIV-AIDS and TB detection test on the theory results. Numerical simulation of the model for various key parameters involved and their impact on the spread of disease is also presented.

**Keywords:** HIV-AIDS Epidemic, TB-Pathogen, Illegal Migrants.

**2010 Mathematics Subject Classification:** Primary 92D30; Secondary 34C60, 34D20.

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## A CONTROLLABILITY ELUCIDATION- IMPULSIVE FRACTIONAL HIGHER ORDER NEUTRAL DELAY DIFFERENTIAL SYSTEMS

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✓ ☒ Oral presentation      ☐ Poster Presentation

### Abstract:

Aforementioned manuscript is flustered with a controllability interpretation of higher order impulsive fractional neutral type state delay differential systems. By employing Laplace transformation technique and using Mittag-Leffler function, solutions are acquired for the examined fractional delay differential equations. The necessary and sufficient conditions are procured by utilizing algebraic approach method. Eventually, two computative examples are given to manifest the validity of the obtained theoretical results.

**Keywords:** State Delay; Controllability; Mittag-Leffler Function.

**93B05, 34A08:**

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## FIXED POINT THEOREM APPLICATIONS IN COMBATING MISINFORMATION SPREAD THROUGH SOCIAL MEDIA

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### **Abstract:**

Misinformation flow on social network is rising rapidly and it causes many problems. Some of them are: manipulation issues, strong influence in shaping wrong public opinion, political influence through fake news, and many other problems which arise daily. This paper reveals the new approach for analyzing misinformation spread through social media. The research shows how Brouwer Fixed-Point Theorem and Sperner's Lemma can contribute to social networks problem solving. The main focus is on influential users which can spread the misinformation in the short period of the time to the big number of social media users. The goal is to help the process of designing technical and human systems that can minimize the spread of lies and fake informations.

**Keywords:** Misinformation, Social Media, Fixed Point Theorem

**2010 Mathematics Subject Classification:** Applied Mathematics

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A MATHEMATICAL APPROACH FOR TURBULENCE AND ENSTROPY IN THE  
TAYLOR GREEN VORTEX MODEL

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**Abstract:**

The time series expansion of the Euler equation for the unsteady three dimensional flow of an inviscid, incompressible, of Taylor' model, are obtained to 10 terms exactly by means of symbolic calculations and compared well with results of ref [1]. Using a classic example proposed by G. I. Taylor, we are using the computer algebra, the mathematical analysis of a fundamental process in turbulent flow, namely: how do large scale eddies evolve into smaller scale ones. The explicit symbolic series solution of this problem, even for cleverly chosen special cases, requires daunting algebra, and so computational methods have become quite popular. With the aid of a computer algebra system, we have found Taylor and Green's results and obtained more detailed time-series. We have extended their approximation of the energy dissipation from order 5 in time to order 7. Although the range of the radius of convergence is small but pade approximation leads our result to be good even for higher value of the time Pade approximation leads our result to be good even for much higher value of the time namely around 1. It seems that the series has random sign patterns. That means the nearest singularity is somewhere in complex plane of time. The aim is to sum the series and for that Pade approximants has been used in original forms to enable us to increase the range of applicability of the series as has been used in the works of Mansour [2] and Vandyke [3].

**Keywords:** nonlinear differential equations, computer algebra, singularities.

**2010 Mathematics Subject Classification:** Differential Equations

**References**

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## RECENT DEVELOPMENTS ON FULLY FUZZY LINEAR PROGRAMMING

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**Abstract:** Fuzzy numbers are strong tools to represent vagueness in real numbers, which we face in real life expressions such as: approximately or almost. Many types of fuzzy numbers are applied to many mathematical fields in the literature. Recently, Tuffaha and Alrefaie [1] proposed convenient arithmetic operations on the n-polygonal fuzzy numbers, which generalizes some of the widely used types of fuzzy numbers, such as the triangular and the trapezoidal fuzzy numbers. The arithmetic operations were shown to satisfy the most important properties [2]. In this paper, we use this fuzzy number to represent fuzziness of the fully fuzzy linear programming (FFLP), where all the attributes and variables of the problem are polygonal fuzzy numbers. The problem is solved by a fuzzy version of the simplex method, which is shown to generalize the conventional simplex method. We also discuss two methods to start a fuzzy basic feasible solution, these are generalizations of the Big-M and the 2-phase methods. The proposed method is compared with other solution methods of the FFLP problems including the methods of Kumar and Kaur [3], Das et. al [4], and Ganesan and Veeramani [5]. The results show that the proposed method is more realistic than the other three methods since the solution coincides with the solution of the crisp version of the problem, while this doesn't hold in the other methods.

**Keywords:** Fully Fuzzy Linear Programming; Fuzzy Numbers; Fuzzy Simplex.

**2010 Mathematics Subject Classification:** 90C70

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## ON MS-STABILITY OF STOCHASTIC DIFFERENTIAL SYSTEMS

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**Abstract:** Particular criteria of MS-stability for two-dimensional stochastic differential systems given by the authors in [4] are here applied to investigate the stability behavior of systems driven by non-normal drift coefficients under the influence of different kind of noises. Non-normality effects in deterministic systems have been widely studied, but only few recent works analyze the behavior of stochastic systems obtained perturbing such non-normal systems by a multiplicative noise, see e.g. [3]. In [1], using some results that appear in [2], systems with non-normal drift coefficient perturbed by different geometrical noises are studied. The main results are given as conjectures and MS-stability behavior of the systems are shown by means of some pictures. Here we complete this analysis, stating and proving such conjectures. Explicit necessary and sufficient conditions in terms of the coefficients of the systems for MS-stability are given in closed form. In addition,  $\theta$ -methods are applied to solve numerically the SDEs and the MS-stability region of each method is compared with the MS-stability region of test equation.

**Keywords:** Stochastic differential systems, mean square stability, non-normality

**2010 Mathematics Subject Classification:** 60H05

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## DISCRIPTION OF GROUND STATE OF $\lambda$ -MODEL ON THE CAYLEY TREE

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☒ Oral presentation      ☐ Poster Presentation

**Abstract:** It is known that statistical mechanics is interested with the average properties of a mechanical system. Some examples are like the water in a kettle, the atmosphere inside a room and the number of atoms in a magnet bar. These kinds of systems are made up of a large number of components, usually molecules. The observer has restricted power to consider all the component. All we can do is to specify a few average quantity of the system such as its density, pressure or temperature. The main objective of statistical mechanics is to predict the relation between the observable macroscopic properties of the system, given only a knowledge of the microscopic interactions between the components. The present paper is devoted to a model whose interacting molecules are located on nearest neighbor vertices of a Cayley tree. In this paper, we investigate ground states and Gibbs measures of  $\lambda$ -model on a Cayley tree of order two. This investigation is closely related to the phase transitions phenomenon for lattice models on trees. We consider the model where spin have only three values. For this kind of model, we are going to describe all its ground states and study phase transition phenomena by means of Gibbs measures.

**Keywords:** Mathematical Physics, Statesical Mechanics, Cayley Tree.

**2010 Mathematics Subject Classification:** Applied Mathematics

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## Fractional Optimal Rearrangement Problems

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### Abstract:

Let  $D$  be a bounded open set in  $R^n$ . We consider the minimization and the maximization of the functional

$$\Phi_s(f) = \frac{1}{2} \int \int_{R^{2n}} \frac{(u_f(x) - u_f(y))^2}{|x - y|^{n+2s}} dx dy,$$

where  $u_f$  is the unique solution of the equation

$$(-\Delta)^s u = f \text{ in } D,$$

$$u = 0 \text{ in } D^c,$$

over the convex closed set

$$\{f \in L^\infty(D) : 0 \leq f \leq 1, \int_D f dx = \beta\}.$$

We show the existence of the unique minimizer  $f_{min}$  and a maximizer  $f_{max}$ .

Moreover, for some constants  $\alpha_{min}$  and  $\alpha_{max}$  the functions  $u_{min} = \alpha_{min} - u_{f_{min}}$  and  $u_{max} = u_{f_{max}}$  solve the following equations in  $D$

$$-(-\Delta)^s u_{min} - \chi_{\{u_{min} \leq 0\}} \min\{-(-\Delta)^s u_{min}^+; 1\} = \chi_{\{u_{min} > 0\}},$$

$$(-\Delta)^s u_{max} = \chi_{\{u_{max} > \alpha_{max}\}}.$$

**Keywords:** fractional PDEs, rearrangement problems, free boundary

**2010 Mathematics Subject Classification:** 35R11, 35J60, 35R35

# Designs, Codes and Graphs

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Special Session C1: Designs, Codes and Graphs Chair: Maheshanand Bhaintwal Room:			
Time	ID	Title	Speaker
10:30-10:55	AA-3049	Locally Recoverable Codes with Intersecting Recovering Sets	Maheshanand Bhaintwal
10:55-11:20	AA-3020	The Graphs Cospectral with the Pineapple Graph	Hatice Topcu
11:20-11:45	AA-3046	Vandermonde Sets and Hyperovals	Duy Ho
11:45-12:10	AA-3059	Hyperovals and Bent Functions	Kanat Abdukhalikov
12:10-12:35	AA-3043	$\mathbb{Z}_2\mathbb{Z}_4$ -Additive Quadratic Residue Codes	Taher Abualrub

## LOCALLY RECOVERABLE CODES WITH INTERSECTING RECOVERING SETS

Charul Rajput, Maheshanand Bhaintwal

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**General area of research:** Algebra and Its Applications

### Abstract:

Locally recoverable (LRC) codes came into prominence due to their applications in data storage systems. They are frequently used in distributed storage systems and cloud storage systems. A common scenario in these systems is the failure of a single node. LRC codes efficiently handle the problem of recovering a failed node. A code is called an LRC code if the value at the  $i$ th coordinate position of any codeword can be recovered by accessing a small number of other coordinate positions. The set of such coordinate positions is called a *recovering set* for the  $i$ th coordinate position.

LRC codes were introduced by Gopalan et al. [1]. Tamo and Barg [3] have given a construction of LRC codes which is based on the classical construction of Reed-Solomon (RS) codes. These codes are called RS-like LRC codes. LRC codes with availability is a generalization of LRC codes, in which there are more than one recovering set for each coordinate position, and it is generally assumed that these recovering sets are pairwise disjoint. The availability feature costs an increment in the length of the code. Recently Kruglik et al. [2] have given a further generalization of these codes in which the recovering sets need not be disjoint; they can have a small number of elements in common. Since recovering sets have small number of common elements, the code is still suitable for load balancing problem. The benefit of adding this feature is that the achievable code rate can be increased. In this talk, we present a construction of such type of codes by using the construction of RS-like LRC codes. We also discuss a bound on the rate of the codes from this construction, and present a sufficient condition for a cyclic code over a finite field to be an LRC code with intersecting recovering sets.

**Keywords:** LRC codes, availability, intersecting recovering sets

**2010 Mathematics Subject Classification:** 94B05, 94B15, 94B60

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## The Graphs Cospectral with the Pineapple Graph

**Hatice Topcu**

Nevsehir Hacı Bektas Veli University, TURKEY, *haticekamittopcu@gmail.com*

### **Abstract:**

The pineapple graph  $K_{p,q}$  is obtained by appending  $q$  pendant edges to a vertex of a complete graph  $K_p$  ( $q \geq 1, p \geq 3$ ). Firstly, Zhang and Zhang (2009) [1] claim that the pineapple graphs are determined by their adjacency spectrum. But Topcu, Sorgun and Haemers (2016) [2] showed that their claim is actually false by constructing graphs which are cospectral and non-isomorphic with  $K_{p,q}$  for every  $p \geq 4$  and various values of  $q$ . In addition they have also proved that the claim is true if  $q=2$ , and referred to the literature for  $q=1, p=3$ , and  $(p,q)=(4,3)$ . By the help of the mixed extension concept that is defined by Haemers (2019) [3], it is proven that pineapple graphs are determined by their spectrum among connected graphs (2019) [4]. Also, in the same paper all disconnected graphs which are cospectral with the pineapple graph are determined. Hence spectral characterization of the pineapple graph was completed. In here, we talk about all of these results about the pineapple graph.

**Keywords:** pineapple graph, adjacency matrix, spectral characterization

**2010 Mathematics Subject Classification:** 05C50

### **References**

- [1] X. Zhang, H. Zhang, Some graphs determined by their spectra, *Linear Algebra Appl.*, 431 (2009) 1443-1454.
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- [3] W.H. Haemers, Spectral characterization of mixed extensions of small graphs, *Discrete Math.* 342 (2019) 2760-2764.
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## VANDERMONDE SETS AND HYPEROVALS

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**Abstract:** In a finite projective plane of order  $q$ , a *hyperoval* is a set of  $q + 2$  points no three of which are collinear. We consider Vandermonde sets and their connections with hyperovals and describe a criteria for a finite set to be a hyperoval in terms of power sums. We also review different ways to represent hyperovals with polynomials and provide new characterisations of hyperovals in terms of these polynomials. Connections with bent functions and MDS-codes will also be discussed.

**Keywords:** hyperoval, finite geometry, projective plane, bent functions

**2010 Mathematics Subject Classification:** 51E15, 51E21, 05B25, 94A60.

## HYPEROVALS AND BENT FUNCTIONS

Kanat Abdukhalikov

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### Abstract:

Niho bent functions are in one-to-one correspondence with line ovals in an affine plane. Points of the line oval completely define the dual bent function. Furthermore, Niho bent functions are in one-to-one correspondence with ovals in a projective plane  $PG(2, q)$  with nucleus at a designated point [1]. Any oval can be obtained from a hyperoval by removing one point. We determine the equivalence classes of Niho bent functions corresponding to a hyperoval and describe the equivalence classes of Niho bent functions for all known types of hyperovals [2].

**Keywords:** Hyperovals, line ovals, bent functions.

**2010 Mathematics Subject Classification:** 51E15, 51E21, 51E23, 94A60.

### References

- [1] Hyperovals and bent functions, European Journal of Combinatorics 79 (2019), 123–139.
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## $Z_2Z_4$ -Additive Quadratic Residue Codes

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<sup>b</sup>Department of Mathematics and Statistics, American University of Sharjah, Sharjah, UAE.

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### Abstract:

In this paper, we are interested to study the structure and properties of quadratic residue codes (QRC) over the ring  $Z_2Z_4$  of length  $n=p+q$  with primes  $p$  and  $q$  satisfy  $p \equiv \pm 1 \pmod 8$  and  $q \equiv \pm 1 \pmod 8$ . We will find the structure and properties of separable and non-separable QRC over  $Z_2Z_4$ . In particular, we will find the idempotent generators for these codes. We will also construct a family of self-dual and formally self-dual codes derived from QRC over  $Z_2Z_4$ . Examples of QRC over  $Z_2Z_4$  of different lengths will be constructed.

**Keywords:** Additive codes, Quadratic residue codes (QRC), Separable and non-separable QRC.

**2010 Mathematics Subject Classification:** 94B05, 94B15.

### References

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# Discrete Dynamic Modeling of Biological Systems

*Special Session Organized by Abdul Jarrah and Fillippo Castiglione*

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Special Session A1: Discrete Dynamic Modeling of Biological Systems Chair: Abdul Jarrah Room:SBA0006			
Time	ID	Title	Speaker
14:00-14:30	MB-3004	A Multiscale Model of the Innate Immune Response to Respiratory Fungal Infections	Reinhard Laubenbacher
14:30-15:00	MB-3009	Deciphering Yeast Physiology by a Multi-Scale Framework Integrating Cell Cycle and Metabolism	Matteo Barberis
15:00-15:30	MB-3015	Compositionality in the Boolean Model of Regulatory Networks	Marco Pedicini

Special Session A2: Discrete Dynamic Modeling of Biological Systems Chair: Fillippo Castiglione Room:SBA0006			
Time	ID	Title	Speaker
16:00-16:30	MB-3001	Network Control through Multistate Canalization	Elena Dimitrova
16:30-17:00	MB-3065	Network Reconstruction using Computational Algebra and Gene Knockouts	Matthew Macauley
17:00-17:30	MB-3002	Methylation Challenges & Opportunities for Biomarkers Identification – Focus on Imputation	Christine Nardini
17:30-18:00	MB-3006	Critical Nodes Reveal Remarkable Features of Human Essential Genes and Protein Interactome	Paolo Tieri

## A MULTISCALE MODEL OF THE INNATE IMMUNE RESPONSE TO RESPIRATORY FUNGAL INFECTIONS

**Reinhard Laubenbacher**

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### **Abstract:**

Invasive aspergillosis is among the most common fungal infections in immunocompromised hosts and carries a poor outcome. The spores of the causative organism, *Aspergillus fumigatus*, are ubiquitously distributed in the environment. Healthy hosts clear inhaled spores without developing disease, but individuals with impaired immunity are susceptible to a life-threatening respiratory infection that can then disseminate to other organs. The increasing use of immunosuppressive therapies in transplantation and cancer has dramatically increased suffering and death from this infection, and this trend is expected to continue. Current therapeutic approaches have been focused primarily on the pathogen, but a better understanding of host defenses in this infection may lead to the development of new treatments. This talk presents a multi-scale mathematical model that can serve as a simulation tool of the innate immune response to invasive aspergillosis, and the exploration of host-centric therapeutic approaches.

**Keywords:** multiscale mathematical model, fungal pathogen, immune response.

**2010 Mathematics Subject Classification:** 92

## DECIPHERING YEAST PHYSIOLOGY BY A MULTI-SCALE FRAMEWORK INTEGRATING CELL CYCLE AND METABOLISM

Lucas van der Zee<sup>1,2</sup>, Edoardo Saccenti<sup>3</sup>, Thierry D.G.A. Mondeel<sup>1,2</sup>, Kate Campbell<sup>4</sup>,  
Hans V. Westerhoff<sup>2</sup>, Jens Nielsen<sup>4,5</sup>, and **Matteo Barberis**<sup>1,2,\*</sup>

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<sup>2</sup>Synthetic Systems Biology and Nuclear Organization, University of Amsterdam, Amsterdam, The Netherlands

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### Abstract:

Cell division cycle and metabolism are coupled networks. Cell growth and division require synthesis of macromolecules which is dependent on metabolic cues. Conversely, metabolites involved in storage metabolism fluctuate periodically during cell cycle progression. High-throughput and mechanistic interactions are reported between these two networks, and computer models of cell cycle and metabolism are being developed. However, to date no effort has been made to explore how cell's physiology is regulated by the integration of these networks in any organism.

Here a multi-scale framework is presented that integrates a Boolean cell cycle model with a constraint-based model of metabolism in budding yeast. An evolutionary optimization algorithm has been developed to generate models that incorporate mechanistic and high-throughput interactions iteratively, to explore their directionality and effect. Random sets of interactions are optimized by the evolutionary algorithm and used as baseline, and model results are verified against metabolic pathway activity and enzyme concentrations.

The real set of interactions showed higher scores with respect to the proteomic data and a consistent interaction pattern. Through machine learning, relevant interactions between cell cycle and metabolism were identified, and design criteria of cell cycle-mediated metabolic regulations were predicted that affect definite cell cycle phases.

The first multi-scale framework that integrates cell cycle and metabolism in budding yeast reveals marked changes in flux distributions through different cell cycle phases. This framework may capture the mechanistic basis of robustness of cell cycle networks.

## Compositionality in the boolean model of regulatory networks

Marco Pedicini

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**Abstract:** A gene regulatory network (GRN) is a static representation of the mutual influence among genes and can be used to simulate the dynamics of the activation/inhibition gene patterns. Under the perspectives of network science and systems biology, the characterization of transcriptional regulatory networks (TRN) beyond the context of model organisms offers a versatile tool whose potential remains yet mainly unexplored. Simulation of immune response is a complex combination of many components and it is modelled in the setting of the agent-based model. An important question is how to combine the cell internal machinery (expressed as a GRN or TRN) to write rules at the mesoscopic level.

In a recent work, [SNA<sup>+</sup>11] gave an updated version of the TR network of *Mycobacterium tuberculosis* (M.tb), which incorporates newly characterized transcriptional regulations corresponding to 30% of the entire genome of the bacterium. Interleaving ergodic properties of the transcriptional modulation of the bacterial and human immune system response in a whole model is a completely new and difficult approach. Because of the large number of genes involved, novel algorithms based on formal methods are required to increase our understanding of how network actions can be translated and make explicit in the rules incorporated in simulation of its dynamics.

This is an example of a more general question: how to predict the effect on dynamics when composition operations are applied to regulatory networks. The development of a library of methods which permit to combine networks and which is sound concerning dynamical invariants, would be of great impact on the difficult and key task of inferring GRN from datasets in systems biology, particularly in the case of large networks, [CMK17].

**Keywords:** Gene Regulatory Networks, Boolean Networks, Synchronous and Asynchronous Dynamics, Compositionality.

**2010 Mathematics Subject Classification:** 92C42, 06E30, 90B10, 94C10, 92D10, 92E10

## References

- [CMK17] Clément Carré, André Mas, and Gabriel Krouk. Reverse engineering highlights potential principles of large gene regulatory network design and learning. *npj Systems Biology and Applications*, 3(1), dec 2017.
- [CMPJ18] Filippo Castiglione, Emiliano Mancini, Marco Pedicini, and Abdul Salam Jarrah. *Quantitative Modelling Approaches*, volume 2. Elsevier Ltd., 2018.
- [PBC<sup>+</sup>10] M. Pedicini, F. Barrenäs, T. Clancy, F. Castiglione, E. Hovig, K. Kanduri, D. Santoni, and M. Benson. Combining network modeling and gene expression microarray analysis to explore the dynamics of Th1 and Th2 cell regulation. *PLoS Computational Biology*, 6(12), 2010.
- [PPC18] M. Pedicini, M.C. Palumbo, and F. Castiglione. *Computing hierarchical transition graphs of asynchronous genetic regulatory networks*, volume 830. 2018.
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- [SPC08] Daniele Santoni, Marco Pedicini, and Filippo Castiglione. Implementation of a regulatory gene network to simulate the TH1/2 differentiation in an agent-based model of hypersensitivity reactions. *Bioinformatics*, 24(11):1374–1380, 2008.

## NETWORK CONTROL THROUGH MULTISTATE CANALIZATION

Elena Dimitrova

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Obispo, CA 93407, USA, *edimitro@calpoly.edu*

### **Abstract:**

Boolean canalization, a type of hierarchical clustering of the inputs of a Boolean function, has been studied in the context of network modeling where each layer of canalization adds a degree of stability in the dynamics of the network. Multicellular populations give rise to emergent features such as patterns based upon the collective communication between neighboring and distant cells. This talk will present a recently introduced generalization of canalization to multistate functions and discuss the role of canalization in the study and control of multicellular populations.

**Keywords:** Boolean Functions, Control, Canalization

**2010 Mathematics Subject Classification:** 92B05

## Network reconstruction using computational algebra and gene knockouts

Matthew Macauley

School of Mathematical and Statistical Sciences, Clemson University, Clemson, South Carolina, USA.  
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### **Abstract:**

I will discuss an ongoing project to reconstruct a gene network from time-series data from a mammalian signaling pathway. The data is generated from gene knockouts and the techniques involve computational algebra. Specifically, one creates an pseudomonomial “ideal of non-disposable sets” and applies an analogue of Stanley-Reisner theory and Alexander duality to it. Of course, things never work as well in practice, due to issues such as noise, discretization, and scalability, and so I will discuss some of these challenges and current progress.

**Keywords:** Algebraic geometry, biological networks, reverse engineering.

**2010 Mathematics Subject Classification:** 92C42, 05E40, 68Q80

## METHYLATION CHALLENGES & OPPORTUNITIES FOR BIOMARKERS IDENTIFICATION – FOCUS ON IMPUTATION

Pietro Di Lena <sup>1</sup>, Claudia Sala <sup>2</sup>, Andrea Prodi <sup>3</sup> and Christine Nardini <sup>4</sup>

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<sup>3</sup>Smart Cities Living Lab, Institute of Organic Synthesis and Photoreactivity, CNR, Bologna, Italy;  
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### Abstract:

Physiological (development, aging) and pathological conditions (autoimmune maladies, cancers and other numerous diseases) are strongly influenced by DNA methylation, a stable epigenetic alteration occurring in the cells' nucleus, during an individual's life. In particular, ample part of the research in methylation focuses on the development of molecular age estimation methods based on DNA methylation levels (mAge [1]), as a potential early marker of diseases. In fact, large numbers of studies indicate that divergences between mAge and chronological age may be a powerful indicator of non-physiological conditions [2]. This research has been boosted by the evolution of high-throughput technologies enabling the quantification of DNA methylation levels across the human genome.

Several mechanisms still need to be elucidated, yet the peculiar biochemistry of the phenomenon can be used as a base to enhance current approaches of analysis. In particular, estimation of mAge can be impaired by multiple missing values. Although several imputation methods exist, a major deficiency lies in the inability to cope with large datasets, such as DNA methylation chips.

We present a simple and computationally efficient imputation method, *methyLImp*, based on linear regression [3]. The rationale of the approach lies in the observation that methylation levels show a high degree of inter-sample correlation. We performed a comparative study of our approach with other imputation methods on DNA methylation data of healthy and disease samples from different tissues. Performances have been assessed both in terms of imputation accuracy and in terms of mAge estimation. Our linear regression model proves to perform equally or better in terms of accuracy with better computational efficiency.

Further we highlight future directions and potential applications that may benefit from the preservation of datasets wholeness, better granted by data imputation.

**Keywords:** methylation, imputation, mAge.

**2010 Mathematics Subject Classification:** 92B05 General biology and biomathematics

### References

- [1] S. Horvath, "DNA methylation age of human tissues and cell types," *Genome Biol.*, vol. 14, no. 10, p. R115, 2013.
- [2] C. Nardini, J.-F. Moreau, N. Gensous, F. Ravaioli, P. Garagnani, and M. G. Bacalini, "Seminars in Immunology," *The epigenetics of inflammaging – heterochromatin loss, gene-specific remodelling, environmental stimuli*, 2018.
- [3] P. Di Lena, C. Sala, A. Prodi, and C. Nardini, "Missing value estimation methods for DNA methylation data," *Bioinforma. Oxf. Engl.*, vol. 35, no. 19, pp. 3786–3793, Oct. 2019.

## CRITICAL NODES REVEAL REMARKABLE FEATURES OF HUMAN ESSENTIAL GENES AND PROTEIN INTERACTOME

Alessandro Celestini<sup>1</sup>, Marco Cianfriglia<sup>1</sup>, Enrico Mastrostefano<sup>1</sup>, Alessandro Palma<sup>2</sup>, **Paolo Tieri**<sup>1</sup>

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<sup>2</sup> Molecular Genetics Laboratory, University of Tor Vergata, Rome, Italy

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### Abstract:

Network-based ranking methods (e.g., centrality analysis) have found extensive use in systems biology and network medicine for the prediction of essential proteins, for the prioritization of drug targets candidates in the treatment of several pathologies and in biomarker discovery, and for human disease genes identification [1].

We here study the structure and the connectivity of the human protein-protein interaction network (*a.k.a.* the *interactome*) to find the nodes whose removal has the heaviest impact on the network, *i.e.* that maximizes its fragmentation. Such nodes are known as Critical Nodes (CNs). Specifically, we implemented a Critical Node Heuristic (CNH) and compared its performance against other four heuristics based on well known centrality measures [2, 3]. To better understand the structure of the interactome, the CNs' role played in the network, and the different heuristics' capabilities to grasp biologically relevant nodes, we compared the sets of nodes identified as CNs by each heuristic with two experimentally validated sets of essential genes, *i.e.* the genes whose removal impact on a given organism's ability to survive [4].

Our results show that classical centrality measures (*i.e.* closeness centrality, degree) found more essential genes with respect to CNH on the current version of the human interactome, however removing these nodes do not have the greatest impact on interactome connectivity, while, interestingly, the genes identified by CNH show peculiar characteristics both from the topological and the biological point of view. Finally, even if a relevant fraction of essential genes is found via the classical centrality measures, the same measures seem to fail in identifying the whole set of essential genes, suggesting once again that some of them are not central in the network, that there might be biases in interaction data, and that different, combined graph theoretical and other techniques should be applied for their discovery.

**Keywords:** critical node, human interactome, connectivity.

**2010 Mathematics Subject Classification:** 92C42, 97M60

### References

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- [2] Paudel N, Georgiadis L, Italiano GF. Computing critical nodes in directed graphs. *Journal of Experimental Algorithmics (JEA)*, 23(2):2–2, 2018.
- [3] Liu X, Hong Z, Liu J, Lin Y, Rodríguez-Patón A, Zou Q, Zeng X. Computational methods for identifying the critical nodes in biological networks. *Brief Bioinform.* 2019
- [4] Boone C, Andrews BJ. The indispensable genome. *Science*, 350(6264):1028–1029, 2015.

# Discrete Mathematics

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Session C2: Discrete Mathematics Chair: Mustapha Aouchiche Room:SBA0006			
Time	ID	Title	Speaker
14:00-14:25	DM-3001	On the soEnergy of Graphs	Kahraman Birgin
14:25-14:50	DM-3002	The Edge Metric Dimension of some Subdivisions of the Wheel Graph	M.S. Bataineh
14:50-15:15	DM-3003	Nordhaus–Gaddum Inequality for the Spectral Radius of a Graph	Mustapha Aouchiche
15:15-15:40	DM-3004	On Partition Dimension of Infinite Graphs	Muhammad Imran

## ON THE soENERGY OF GRAPHS

**Kahraman Birgin**, Sezer Sorgun

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**Abstract:** The number of papers concerned with various graph energies in the 21st century, In 2017 Ivan Gutman and Boris Furtula published a paper about survey of graph energies [1]. They mentioned 63 different type of energy in their paper. One of them is soEnergy.

In this paper, we obtained some results on the soEnergy of the special graphs, such as kite, wheel, lollipop etc. Also we have corrected some results on the soEnergy related to the path, cycle and complete graph, given in [2,3].

**Keywords:** Graph, Dominating set, soEnergy.

**2010 Mathematics Subject Classification:** 05C50

### References

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- [3] S. P. Jeyakokila, P. Sumathi, A note on soEnergy of Cocktailparty and Crown Graphs, International Journal of Applied Science and Mathematics. 3 (2016) 2394-2894.

## THE EDGE METRIC DIMENSION OF SOME SUBDIVISIONS OF THE WHEEL GRAPH

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### Abstract:

The aim of this study is to determine the edge metric dimension of some subdivision of the wheel graphs. In particular, we determine and compare the metric and edge metric dimensions of the graphs obtained after the cycle, spoke and barycentric subdivisions of the wheel graph. Furthermore, some families of graphs have been constructed through subdivision process for which  $\dim(\mathcal{WCP}_{n,1}) < \dim_e(\mathcal{WCP}_{n,1})$ , and also  $\dim(\mathcal{WCY}_{n,1}) = \dim_e(\mathcal{WCY}_{n,1})$  which answer a question in [8].

**Keywords:** Edge metric dimension; Subdivision; Wheel graph.

**2010 Mathematics Subject Classification:** 05C12, 05C76, 05C90.

## NORDHAUS–GADDUM INEQUALITY FOR THE SPECTRAL RADIUS OF A GRAPH

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**Abstract:** The spectral radius  $\lambda(G)$  of a graph is the largest eigenvalue of its adjacency matrix. Nordhaus–Gaddum inequalities for a given graph invariant  $\mu$  are of the form  $m(n) \leq \mu(G) + \mu(\overline{G}) \leq M(n)$  and/or  $l(n) \leq \mu(G) \cdot \mu(\overline{G}) \leq L(n)$ , where  $\overline{G}$  is the complement of  $G$  and  $m(n), l(n), M(n), L(n)$  are functions in the order  $n$  of  $G$ . The problem of finding Nordhaus–Gaddum type inequalities for the spectral radius of a graph goes back to the early seventies when Amin and Hakimi (1972) and Nosal (1970), independently, proved lower and upper bounds on the sum of the spectral radii of a graph and its complement. Since then, several such bounds were proven, however the problem of finding a tight upper bound remains open. In this talk, we present a few of those bounds with a focus on the most recent one. We also provide a conjectured tight bound.

**Keywords:** Nordhaus–Gaddum inequalities, Spectral Radius, Graph

**2010 Mathematics Subject Classification:** 05C50

## References

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## ON PARTITION DIMENSION OF INFINITE GRAPHS

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### Abstract:

Little is known about the partition dimension of infinite graphs. Tomescu studied graphs where the set of vertices is the set of points of the integer lattice. We generalise these graphs and present several exact values, lower bounds and upper bounds on the partition dimension of infinite graphs.

**Keywords:** Partition dimension, infinite graph, distance.

**2010 Mathematics Subject Classification:** 05C12

### References

- [1] I. Tomescu, Discrepancies between metric dimension and partition dimension of a connected graph. *Discrete Math.* 308 (2008), 5026–5031.
- [2] I. Tomescu and M. Imran, On metric and partition dimensions of some infinite regular graphs. *Bull. Math. Soc. Sci. Math. Roumanie (N.S.)* 52 (2009), 461–472.

# Financial Mathematics

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Session A1: Financial Mathematics Chair: Mayank Goel Room:SBA0012			
Time	ID	Title	Speaker
14:00-14:25	FM-3002	Causality and Price Discovery of Cross-Listed Nifty Futures	K. Kiran Kumar
14:25-14:50	FM-3006	Risk-Sensitive Benchmarked Portfolio Optimization with General Non-Negative Economic Factors	Mayank Goel
14:50-15:15	FM-3008	Valuation of Volatility Derivatives under Markov Switching Stochastic Volatility Model	Youssef El Khatib

## CAUSALITY AND PRICE DISCOVERY OF CROSS-LISTED NIFTY FUTURES

**K. Kiran Kumar**

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### **Abstract:**

This paper investigates the price discovery and lead-lag relations between National Stock Exchange (NSE) Nifty Index Futures and Singapore exchange (SGX) listed Nifty futures contracts. We use intraday high frequency data of both contracts from July 2016 to June 2017. The daily returns are decomposed into Overlapping and Non-overlapping returns considering the time zone differences between the exchanges. Various competing forms of Vector Auto Regressive (VAR) models along with Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models are employed. The results clearly depict that the SGX listed Nifty Futures have significant power in predicting the overnight returns of NSE Nifty futures, in non-overlapping trading zone context. And during overlapping trading hours, NSE Nifty futures have significant power in explaining the SGX listed Nifty futures contracts and not the other way.

**Keywords:** index futures; GARCH; Granger Causality

**2010 Mathematics Subject Classification:** 91G20; 91G70; 6207

## Risk-sensitive Benchmarked Portfolio Optimization with General Non-negative Economic Factors

Ravi Shankar, and Mayank Goel

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**Abstract:** In this work, we discuss portfolio optimization problem aimed to beat the given benchmark in continuous time under utility framework. In financial markets, mutual funds are direct application for benchmarked assets management problem. In these models, the fund/portfolio is managed to outperform the given market index. Here, the fund performance is evaluated based upon the excess return gained over the benchmark return, which are obtained for minimum risk, and the risk is defined as the variance of excess return. Fundamentally, the choice of benchmark is a reflection of the manager's strategy as well as the investor's objectives and constraints.

The contribution of this paper is to extend the model studied in [1] to solve the risk-sensitive benchmarked assets management problem in the non linear setup. We consider general setting with  $m$  risky assets and  $n$  economic factors, where dynamics of assets and level of economic factors are given by very general non-negative nonlinear stochastic differential equations. In general, economic factors may be restricted to non-negative, such as interest rates and inflation. We approach the solution by investigating the existence and uniqueness of the solution of corresponding Hamilton Jacobi Bellman Equation. Also, this paper provides analytic solution to finite-horizon benchmarked investment problem and then extend the results to consider infinite horizon problem. We finally back test our strategies with market data and conclude the results.

**Keywords:** Portfolio theory; Stochastic control ; Stochastic models.

**2010 Mathematics Subject Classification:** 91G10, 91G80, 91G30

## References

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## VALUATION OF VOLATILITY DERIVATIVES UNDER MARKOV SWITCHING STOCHASTIC VOLATILITY MODEL

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<sup>1</sup>UAE University, Al-Ain, P.O. Box 15551. United Arab Emirates.

<sup>2</sup>CEMOTEV, Universit Versailles Saint-Quentin en Yvelines, Paris-Saclay, France.

<sup>3</sup>Queen's Management School, United Kingdom.

### Abstract:

In this work, a general Markov switching stochastic volatility model with jumps where both the asset and the volatility dynamics depend on the values of a Markov jump process and where jumps can occur in the dynamic. Due to the stochastic volatility, the jump component and the Markov regime switching, this financial market is thus incomplete and perfect pricing and hedging of options are not possible. Under these settings, the paper investigates the valuation of volatility and variance swaps as well as the valuation of variance swaps in the discretely-sampled variance case. Numerical simulations are provided.

**Keywords:** Volatility Swaps, Markov Switching, Heston with Jumps.

**2010 Mathematics Subject Classification:** 91B25, 91G20, 60H07

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<sup>1</sup>The first author would like to express his sincere appreciation to the United Arab Emirates University Research Office for the financial support UPAR Grant No. 31S369.

# Financial Statistics with Application in Cryptocurrency and Blockchain

*Special Session Organized by Stephen Chan*

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Special Session A2: Financial Statistics with Application in Cryptocurrency and Blockchain Chair: Stephen Chan Room:SBA0011			
Time	ID	Title	Speaker
16:00-16:25	FM-3007	The Adaptive Market Hypothesis in the High Frequency Cryptocurrency Market	Jeffrey Chu
16:25-16:50	FM-3004	Trading Models on Cryptocurrencies	Shou Hsing Shih
16:50-17:15	ST-3012	Modelling Cryptocurrencies using Undirected Graphs	Paola Stolfi
17:15-17:40	FM-3003	Flexible Models for Stock Returns based on Student's t-Distribution	Emmanuel Afuecheta

## THE ADAPTIVE MARKET HYPOTHESIS IN THE HIGH FREQUENCY CRYPTOCURRENCY MARKET

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<sup>c</sup> Department of Mathematics and Statistics, American University of Sharjah, PO Box 26666, Sharjah, United Arab Emirates, [schan@aus.edu](mailto:schan@aus.edu)

### Abstract:

This paper investigates the adaptive market hypothesis (AMH) with respect to the high frequency markets of the two largest cryptocurrencies: Bitcoin and Ethereum, versus the Euro and US Dollar. Our findings are consistent with the AMH and show that the efficiency of the markets varies over time. We also discuss possible news and events which coincide with significant changes in the market efficiency. Furthermore, we analyse the effect of the sentiment of these news and other factors (events) on the market efficiency in the high frequency setting, and provide a simple event analysis to investigate whether specific factors affect the market efficiency/inefficiency. The results show that the sentiment and types of news and events may not be a significant factor in determining the efficiency of cryptocurrency markets.

**Keywords:** Bitcoin; Adaptive market hypothesis; Efficient market hypothesis.

### 2010 Mathematics Subject Classification:

### References

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- [4] Nadarajah, S. and Chu, J. 2017. On the inefficiency of Bitcoin. *Economics Letters*, 150, pp. 6-9.

## Trading Models on Cryptocurrencies

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### **Abstract:**

The purpose of the current study is to develop trading models for different cryptocurrencies. Cryptocurrencies are known as highly unpredictable due to their high volatility. Our algorithm is capable of helping traders efficiently identifying the relationships between different pairs of cryptocurrencies, and find those pairs of cryptocurrencies that are worth of trading. Our trading models will be able to identify patterns and make predictions, which will be extremely useful for coin to coin traders. A brief summary of the algorithm is given. To illustrate the quality of our proposed algorithm, we study the pattern of ten different reputable cryptocurrencies and use their daily closing prices to constitute a time series. The comparison between our proposed forecasting algorithm versus the autoregressive integrated moving average (ARIMA) process will be demonstrated.

**Keywords:** Time Series Forecasting, Cryptocurrencies, Polynomial Trend Analysis.

## Modelling cryptocurrencies using undirected graphs

Mauro Bernardi, **Paola Stolfi**, Davide Vergni

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**Abstract:** During the last two years the interest toward cryptocurrencies arose exponentially as well as the number of people investing on them and the online exchange websites offering the opportunity to build cryptocurrencies’ portfolio. However, since cryptocurrencies are independent from central banks and financial institutions, there are no fundamentals supporting portfolio diversification strategy for risk minimisation. Therefore, our interest is to investigate their joint evolution adopting a statistical learning approach to get insights from their time series. In particular, we used undirected graphs, which are useful tools when dealing with high-dimensional dependence structures, properly modified in order to account for the distinctive nature of the data being characterised by the non stationary behaviour and by the presence of extreme values that highly deteriorates signal extraction.

Despite the practical relevance of undirected graphs, most of the recent contributions have been confined under the restrictive assumption of independently and identically distributed Gaussian observations, with only few exceptions (see Lafferty et al. (2012), Finegold and Drton (2014), Vogel and Tyler (2014)). However, they do not deal with the potentially time-varying nature of variance-covariance matrices that may originate from dependent data. This case has been recently investigated by Zhou et al. (2010), where they introduce sparse time-varying undirected graphs, namely, graphs whose structure evolves smoothly over time. These models works well in high dimensional settings, that is when lots of parameters need to be estimated and few observations are available. Unfortunately, the Gaussian assumption makes the resulting estimates quite sensitive to the presence of outliers. The contribution of this paper is to solve this issue by proposing a robustification of the sparse time-varying graphs proposed by Zhou et al. (2010). Specifically, following Hirose et al. (2017), we propose a robust estimator which minimises the  $\gamma$ -divergence between the postulated and theoretical distribution. Furthermore, we provide an algorithm to handle parameter estimation based on the “Expectation-Minimisation” approach.

**Keywords:** Undirected graphs, non stationary time series, robust methods.

**2010 Mathematics Subject Classification:**62

## References

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FLEXIBLE MODELS FOR STOCK RETURNS BASED ON  
STUDENT'S T DISTRIBUTION

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**Abstract:**

Models based on the Student's  $t$  distribution are proposed with its scale parameter randomized. Mathematical properties of the models such as their probability density functions, cumulative distribution functions, moments and characteristic functions are derived. Three of the models are fitted to daily log returns of six financial indices. They were shown to provide better fits than mixtures of Student's  $t$  distributions and the popular generalized hyperbolic distribution.

**Keywords:** Probability Distribution, Stock returns, Value at risk.

**2010 Mathematics Subject Classification:** 62

**References**

[1] Flexible Models for Stock Returns Based on Student's  $T$  Distribution  
The Manchester School 87 (3), 403-427

# Mathematical Biology

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Session C1: Mathematical Biology Chair: Ziyad AlSharawi Room:SBA0006			
Time	ID	Title	Speaker
10:30-10:55	MB-3013	Three Species Predator-Prey Interactions with Historic Behavior	Mansur Saburov
10:55-11:20	MB-3005	Edge Clustering Coefficient (ECC) based Node Centrality	Ahmed Khasim
11:20-11:45	MB-3007	Mathematical Model of Glioma and Chemotherapy	Dua Alahmadi
11:45-12:10	MB-3011	Stability and Bifurcation Analysis of a Discrete-Time Predator-Prey Model with Strong Allee Effect	Ziyad AlSharawi
12:10-12:35	MB-3014	Lens Free High Resolution Computational Imaging using Fourier Techniques	Suhas P. Poyyilveetil

## EDGE CLUSTERING COEFFICIENT (ECC) BASED NODE CENTRALITY MEASURE TO IDENTIFY IMPORTANT GENES IN ALZHEIMER'S DISEASE

Ahamed Khasim K<sup>\*,a</sup>, Ajith K. M.<sup>\*,b</sup>, Suhas P Veetil<sup>†,c</sup>, T. K. Shajahan <sup>\*,d</sup>

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### Abstract:

Alzheimer's disease (AD) is a neuro-degenerative disorder characterised by progressive loss of neurons in the central nervous system. Different hypotheses have been suggested to explain the AD mechanism at molecular level. The main hypothesis attempts to explain the pathogenesis via amyloid  $\beta$  and *tau* protein accumulation. The other hypotheses discuss the role of inflammation, synaptic pathology, and the role of impaired insulin signalization in AD progression[1]. We use a network approach to analyse the protein interaction data associated with the AD and to identify the potential bio-markers. A set of 429 genes associated with Alzheimer's (terminal nodes) is collected by screening publications on genetic association studies deposited in the PubMed[2]. Klein-Ravi algorithm is used to extract a sub-network of AD from a large human protein-protein interaction network (PPI). To find the important nodes, we use a centrality measure based on the edge clustering coefficient (ECC)[3]. This metric calculates the importance of a node based on the number of edges it connects and the edge's clustering coefficient. We identified SYK, CASK, GRB2 and UBC as the important proteins in the network and are not from the terminal nodes.

**Keywords:** Alzheimer's disease, protein-protein interaction network, edge clustering coefficient

**2010 Mathematics Subject Classification:**

### References

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## MATHEMATICAL MODEL OF GLIOMA AND CHEMOTHERAPY

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**General area of research:** Mathematical Biology.

### Abstract:

Glioma is an invasive brain tumor associated with low survival rates and limited life expectancy. Mathematical modeling of glioma allows researchers to test hypotheses that may improve treatment strategies. In this paper, we analyze the mathematical model proposed by Iarosz et al. [10] theoretically and numerically. First, we consider modeling the tumor growth without treatment. The model only describes the competition between glial and glioma cells. Numerical results showed that untreated glioma invades the brain and destroys the glial cells. After that, we consider modeling the tumor growth in which glioma is treated with chemotherapy. The model describes the interactions between glial cells, glioma cells, neurons and the effect of the chemotherapy on these cells. The infusion rate of chemotherapy played a major role in the stability of the equilibria. Finally, we conducted several numerical simulations to support our theoretical findings. The numerical experiments vary according to the values of the infusion rate.

### Keywords:

Mathematical model of brain tumors, Chemotherapy, Stability of equilibria.

### 2010 Mathematics Subject Classification:

92Bxx, 93A30.

## References

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## STABILITY AND BIFURCATION ANALYSIS OF A DISCRETE-TIME PREDATOR-PREY MODEL WITH STRONG ALLEE EFFECT

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### Abstract:

In this talk, we propose a discrete-time predator-prey model with a non-monotonic functional response. The model supports the coexistence of two steady states. We discuss the impact of Allee effect on the stability, then analyze the mathematical features of the model based on local stability and bifurcation theory. By considering the Allee parameter as the bifurcation parameter, we provide sufficient conditions for the period-doubling and the Neimark-Sacker bifurcations. Also, sufficient conditions are given to show that increasing the magnitude of the Allee parameter within a certain domain plays a significant role in stabilizing the system.

**Keywords:** Predator-prey model; Allee effect; Local stability.

**2010 Mathematics Subject Classification:** 92B05 , 39A30, 39A28

### References

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## THREE SPECIES PREDATOR-PREY INTERACTIONS WITH HISTORIC BEHAVIOR

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### Abstract:

The dynamics of the interaction of  $m$  species in population biology and mathematical ecology is frequently described by the discrete-time Kolmogorov system  $\mathcal{K} : \mathbb{R}_+^m \rightarrow \mathbb{R}_+^m$ ,

$$\mathcal{K}(\mathbf{x}) := (x_1 g_1(\mathbf{x}), x_2 g_2(\mathbf{x}), \dots, x_m g_m(\mathbf{x}))$$

where  $x_i$  is a population density of the species  $i$  and the function  $g_i$  is a growth rate of the species  $i$  which depends on the population density vector  $\mathbf{x} = (x_1, x_2, \dots, x_m)$ . Depending on the properties of the functions  $g_i$ , the discrete-time Kolmogorov system represents different kinds of species interactions: (i) *competitive interactions* if the functions  $g_i$  and  $g_j$  are decreasing in  $x_j$  and  $x_i$ , respectively; (ii) *cooperative interactions* if the functions  $g_i$  and  $g_j$  are increasing in  $x_j$  and  $x_i$ , respectively; (iii) *predator-prey interactions* if the function  $g_i$  is increasing in  $x_j$  and the function  $g_j$  is decreasing in  $x_i$ .

The main purpose is to discuss some feature so-called *historic behavior* of a discrete-time Kolmogorov system for three-species predator-prey interactions which causes the non-existence of the time averages  $\frac{1}{n} \sum_{i=0}^{n-1} \mathcal{K}^i(\mathbf{x})$ . The terminology "historic behavior" was coined by Ruelle [1] and the problem of describing the persistent family of dynamical system with historic behavior was popularized by Takens [2, 3].

This work was supported by American University of the Middle East, Kuwait

**Keywords:** Discrete-time Kolmogorov system, predator-prey interaction, historic behavior.

**2010 Mathematics Subject Classification:**

### References

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- [3] F. Takens, Orbits with historic behavior, or non-existence of averages. *Nonlinearity* **21**(3), T33 (2008).

## LENS FREE HIGH RESOLUTION COMPUTATIONAL IMAGING USING FOURIER TECHNIQUES

**Suhas P Veetil**

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### Abstract

A numerical iterative Fourier transform technique is introduced to synthesize a microscopic image using its recorded diffraction pattern. This would replace the use of lenses used in microscopy and hence would provide aberration and label free images. This technique also works for weakly diffracting specimens. Such a Coherent Modulation Imaging Technique(CMT) can bring down the complexity of the experimental setup and additional accessories of conventional microscopy with a numerical process to obtain high resolution imaging. The technique is verified with theoretical simulations and experiments.

**Keywords:** Fourier transform, Coherent diffraction imaging, Ptychography

**2010 Mathematics Subject Classification:** 92B99, 92-08

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# Mathematics Education

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Session C1: Mathematics Education Chair: Saadia Khouyibaba Room:SBA0007			
Time	ID	Title	Speaker
10:30-10:55	ME-3004	How History of Mathematics Can Help in Teaching and Learning Mathematics	Saadia Khouyibaba
10:55-11:20	ME-3003	Concept-Based & Stem-Focused Math Teachers' Preparation Programs	Ali S. Shaqlaih
11:20-11:45	ME-3001	An Ethnomathematics Ios App Designed to Encourage Emirati Grade Six Students to continue taking Mathematics	Jason Johnson

## HOW HISTORY OF MATHEMATICS CAN HELP IN TEACHING AND LEARNING MATHEMATICS

**Saadia Khouyibaba**

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### **Abstract:**

Mathematics.

The foundations of everything we see around us, from the cars we drive to home and work, to the air that we breathe, down to even the sheer concept of time itself. From a young age, we have all learned to accept mathematical concepts as they are; concepts that are seemingly thought of with no process, no train of thought. Students are not given the opportunity to learn why these concepts are the way they are and where they are coming from, they are not given the chance to put themselves in the mind of a mathematician. This is what makes math such a notoriously difficult and *scary* subject. It is crucial that we consider this when teaching this profound subject, to take extra care and always consider these negative connotations associated with it. It is very important to combat these implications, for as long as these intimidating labels are linked to this subject, students will not feel motivated and committed to understanding it.

In this presentation, we will see why and how history of math and its' development through time can be utilized to ease the learning process for the students.

**Keywords:** History of Mathematics, Math Education, Pedagogy

### **2010 Mathematics Subject Classification:**

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## CONCEPT-BASED & STEM-FOCUSED MATH TEACHERS' PREPARATION PROGRAMS

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### **Abstract:**

Mastering Mathematics Content Knowledge is a necessary condition for school teachers to succeed in teaching high school mathematics but of course it is not sufficient. The traditional approach in preparing school teachers has been that pre-service teachers take the math courses (Content Knowledge) in the Mathematics Department and the education courses (Pedagogical Knowledge) in the Education Department. This creates a gap between the advanced math content that teachers learn at college and the curriculum that they will be teaching in their future schools. Even though these teachers' preparation programs require pre-service teachers to take many advanced mathematics courses such as Real Analysis and Abstract Algebra; yet many pre-service teachers don't acquire the conceptual understanding of high school mathematics. This talk will present the primarily results of a study that shows that even though high school math teachers had completed their math requirements, they still can't solve conceptual problems in high school mathematics. For example, all participants were able to find the derivative of a given function, however a few were able to conceptually present the concept of the derivative and come up with some of its applications in their daily life. An alternative approach to teacher preparation program will be introduced.

**Keywords:** Mathematics Content Knowledge, Pedagogical Knowledge, Teacher Preparation Programs

**2010 Mathematics Subject Classification:** 97

## AN ETHNOMATHEMATICS IOS APP DESIGNED TO ENCOURAGE EMIRATI GRADE SIX STUDENTS TO CONTINUE TAKING MATHEMATICS

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### Abstract:

We in the College of Education at Zayed University are committed to excellence. With that in mind, we developed a very ambitious project to create a math app to encourage Emirati students to engage with mathematics, while using Ethnomathematics problems based on the Emirati culture. The Ministry of Education has initiated many advances to improve the teaching and learning of mathematics in the United Arab Emirates. However, The International Mathematics and Science Study (TIMSS 2011), report a different story. TIMSS is a well-respected research report that examines the teaching and learning of mathematics and science around the world (the first report dates back to 1994 and commences every four years). With that said, our project aimed to provide Emirati students (boys and girls) opportunities to make sense of mathematics using ethnomathematics and mobile learning.

We sought to make sure our project was aligned with the UAE educational initiatives. Our project is in line with two indicators in the UAE Vision 2021 National Agenda: **Cohesive Society and Preserved Identity** and **First-Rate Education System**. Our project aimed to honor the Emirati culture and provide students with a sense of pride for their country; by allowing students to see themselves through a mathematics lens (i.e., new math app). Additionally, our project had ambitious aims to provide 6<sup>th</sup> grade Emirati students with a new math app to promote mathematics using examples from the Emirati culture and allow students opportunities to explore learning using smart systems and/or devices. Not only does our project support UAE Vision 2021 National Agenda but also one focus area in the National Science, Technology, and Innovation Policy, which is the **Educational Innovation and Technology**. Our project allowed Emirati students opportunities to make sense of mathematics.

A research project was designed to investigate Emirati students using Mobile Learning to explore Ethnomathematics problems based on the Emirati culture. In other words, such mathematics problems center on the notion on the Emirati culture. A mathematics learning app was developed using these ethnomathematics problems. The learning app allowed students to navigate through four modules to examine various mathematical concepts and provide students with a video, for each module, identifying the historical reference to the UAE. Data for the project were: student completed math problems, survey (student and teacher), and informal interviews (student and teacher). The research team was curious to know how Emirati students solve problems, interact with the app, and accuracy of solutions.

# Number Theory

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Session A1: Number Theory Chair: Armen Bagdsaryan Room:SBA0010			
Time	ID	Title	Speaker
14:00-14:25	NT-3006	On Some Relations Involving Zeros of Riemann Zeta Function and other Zeta Related Functions	Armen Bagdsaryan
14:25-14:50	NT-3001	Analytic Expressions of Characters Defined on Witt Vectors Rings	Siham Mokhfi
14:50-15:15	NT-3008	On Certain Multiple Dirichlet Series of Completely Multiplicative Function	Nabil Tahmi
15:15-15:40	NT-3007	On a Method of Summation of Infinite Series with some Applications to Special Numbers and Zeta Functions	Armen Bagdsaryan

## ON SOME RELATIONS INVOLVING ZEROS OF RIEMANN ZETA FUNCTION AND OTHER ZETA RELATED FUNCTIONS

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**General area of research:** Number Theory, Analysis

### Abstract:

The following results, among others, have been obtained in [1].

**Proposition 1.** *For the Riemann zeta function  $\zeta(s)$ , it holds that*

$$\zeta(k) + \sum_{j=1}^{k-2} \lambda_j \zeta(k-j) + \gamma \lambda_{k-1} + k \lambda_k = 0, \quad (k \geq 2),$$

where  $\lambda_k$  are the coefficients in the Taylor series expansion of  $\tilde{\Gamma}(z) = 1/\Gamma(1-z)$  around  $z = 0$ , and  $\gamma$  is the Euler-Mascheroni constant.

**Proposition 2.** *For the nontrivial zeros  $\rho$  of the Riemann zeta function  $\zeta(s)$ ,*

$$\sum_{j=0}^{k-2} \lambda_j \left( \sum_{\rho} \frac{1}{\rho^{k-j}} \right) + \left[ \frac{1}{2} \gamma + 1 - \log(2\sqrt{\pi}) \right] \lambda_{k-1} + k \lambda_k = 0, \quad (k \geq 2),$$

where  $\lambda_k$  are the coefficients in the Taylor series expansion of the Riemann  $\xi$ -function (or completed zeta function) around  $s = 0$ , and  $\gamma$  is the Euler-Mascheroni constant.

The above propositions have been obtained using several formulas that were derived for the class of entire and meromorphic functions and that relate the sums of the  $n$ th powers of the reciprocals of zeros and poles of these functions with the coefficients of their Taylor series expansions.

In this work I continue my research in this direction and will present some other recurrence formulas for the Riemann zeta function and some other zeta related functions. Certain similar results are expected to be established for the digamma function, Barnes  $G$ -function (or double gamma function), and its logarithmic derivative [2], which will be based on the results of paper [1] and the Weierstrass infinite product representations of entire functions [2, 3, 4]. The determinantal formulas for the sums of the  $n$ th powers of the reciprocals of zeros of entire or meromorphic functions as well as for the coefficients  $\lambda_k$  will also be presented. If time permits, I will discuss some further developments and applications of the same ideas.

**Keywords:** Zeta functions, sums of powers of reciprocals of zeros, entire functions, recurrence formulas

**2010 Mathematics Subject Classification:** 11M06, 30C15, 33B15, 30D99

## References

- [1] A. Bagdasaryan *et al*, Analogues of Newton-Girard power-sum formulas for entire and meromorphic functions with applications to the Riemann zeta function, *J. Number Theory* **147**, 92–102 (2015).
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- [3] I. Mezö, M. Hoffman, Zeros of digamma function and its Barnes  $G$ -function analogue, *Integral Transforms Spec. Funct.* **28**, 846–858 (2017).
- [4] B.Ya. Levin, *Lectures on Entire Functions*. American Mathematical Society: Providence, 1996.

# ANALYTIC EXPRESSIONS OF CHARACTERS DEFINED ON WITT VECTORS RINGS

Siham MOKHFI

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**Abstract:** Let  $p$  be an odd prime number. Let  $F_q$  be a finite field of  $q$  elements of characteristic  $p$ . In our paper[5] we succeeded to establish an analog of Dwork trace formula for Gauss sums on Witt vector rings over  $F_q$  of finite length 2,  $W_2(F_q)$ . This formula was possible thanks to the obtention of analytic expressions for both additive and multiplicative characters of the ring  $W_2(F_q)$ . Here, we will establish an analytic expression of a multiplicative character for the rings of Witt  $W_\ell(F_q)$ . for any integer  $\ell \in N^*$ .

**Keywords:** Splitting Functions, Witt vectors rings.

**2010 Mathematics Subject Classification:**

## References

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- [6] A. PULITA: *Rank one solvable  $p$ -adic differential equations and finite abelian characters via Lubin-Tate groups*. Math. Ann. **337** (2007), no. 3, 489-555.

## On CERTAIN MULTIPLE DIRICHLET SERIES OF COMPLETELY MULTIPLICATIVE FUNCTION

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**Abstract:** Let  $f_r : N^r \longrightarrow C$  be an arithmetic function of  $r$  variables, where  $r \geq 2$ . We study the multiple Dirichlet series defined by

$$D(f_r, s_1, \dots, s_r) = \sum_{n_1, \dots, n_r=1}^{\infty} \frac{f_r(n_1, \dots, n_r)}{n_1^{s_1} \dots n_r^{s_r}}.$$

where  $f_r(n_1, \dots, n_r) = f(n_1)f(n_2) \dots f(n_r)$  and  $f$  is a completely or specially arithmetic function of a single variable. We obtain formulas for these series expressed by an infinite product over all prime numbers and the Dirichlet L-functions. The proof use the formula of Eulerian product generalized. In addition, we apply these formulas on the multiple Dirichlet series associated of certain completely multiplicative functions and specially multiplicative functions, and express these series by the Riemann zeta function.

**Keywords:** Completely multiplicative function, multiple Dirichlet series, Eulerian product.

**2010 Mathematics Subject Classification:**11M32, 11M06, 11A25

## References

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## ON A METHOD OF SUMMATION OF INFINITE SERIES WITH SOME APPLICATIONS TO SPECIAL NUMBERS AND ZETA FUNCTIONS

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**General area of research:** Number Theory, Analysis

**Abstract:** We begin by briefly covering basic concepts and general terminology of the summation method [1, 2], which introduces a natural definition for sums of the form  $\sum_{k=a}^b f(k)$  when  $b < a$  or even negative  $b < 0$ . Sums of the form  $\sum_{k=a}^b$  are classically defined only when the number of terms is a positive integer with  $b \geq a$  or infinity. Our method proposes a natural and systematic way to extend this definition for rather arbitrary limits of summation, and in a way that it does not disrupt the finite limit of a series/sequence that is already convergent, thus making the method regular. We use certain summation formulas, such as:

**Theorem 1.** Let  $f(x)$  be a regular function that satisfies the condition  $f(-x) = f(x - \epsilon t)$ ,  $\epsilon = \{0, -1, 1\}$  and  $t \in \mathbb{N}$  is fixed. Then

$$\sum_{k=1}^{\infty} f(k) = \frac{1}{2} \epsilon \sum_{k=\delta}^{t-1+\delta} \left( \lim_{n \rightarrow \infty} f(n - \epsilon k) - f(-\epsilon k) \right) - \frac{1}{2} f(0), \quad \delta = 2^{-1}(1 - \epsilon).$$

We present several identities, congruences, and recurrence formulas involving Bernoulli and Euler numbers and polynomials, and symmetric polynomials and integer sequences. For instance, we have

**Theorem 2.** Let  $f(x) = \sum_{u=1}^{2k} b_u x^u$  be a polynomial of even degree such that  $f(-x) = f(x - \epsilon t)$ ,  $\epsilon = \pm 1$ ,  $t \in \mathbb{N}$  is fixed. Then

$$\sum_{u=1}^k \frac{b_{2u-1}}{u} B_{2u} = -\epsilon \int_{-1}^0 \left( \sum_{u=\delta}^{t-1+\delta} (f(x - \epsilon u) - f(-\epsilon u)) \right) dx,$$

where  $\delta = (1 - \epsilon)/2$ .

**Proposition 3.** For any prime number  $p > 2$  and odd  $\theta$ ,  $1 < \theta \leq p$ , the congruence holds

$$\theta B_{\theta-1} \equiv - \sum_{u=0}^{\theta-2} S_u^p \binom{\theta}{u} \pmod{p},$$

where  $S_u^p = \frac{1}{p} \sum_{j=0}^{p-1} j^u$ . In particular,  $p B_{p-1} \equiv -1 \pmod{p}$ .

Several new properties of divergent series, analogous to those known for convergent series, are established and then used to obtain closed form evaluations of series involving Riemann's zeta and related functions, including certain zeta related divergent series, and series involving Bernoulli numbers.

**Keywords:** Summation, infinite series and products, Bernoulli and Euler numbers, zeta functions

**2010 Mathematics Subject Classification:** 40C99, 40A25, 11B68, 11M06, 26A03

## References

- [1] Armen Bagdasaryan, An elementary and real approach to values of the Riemann zeta function, *Phys. Atom. Nucl.* **73**, 251–254 (2010).
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# Numerical Analysis

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<b>Session B1: Numerical Analysis</b> <b>Chair: Rama Bhargava</b> <b>Room:SBA0005</b>			
<b>Time</b>	<b>ID</b>	<b>Title</b>	<b>Speaker</b>
9:00-9:25	NA-3002	Instability Boundaries of Double-Diffusive Convection in a Brinkman Bidisperse Porous Medium with an anisotropic Permeability Effect	Sara Saleh
9:25-9:50	NA-3009	The Best Known Interior Point Algorithm for Linear Optimization Problem	El Amir Djaffal
9:50-10:15	NA-3008	A Discontinuous Galerkin Method for Systems of Stochastic Differential Equations with Applications to Population Biology, Finance, and Physics	Helmi Temimi
10:15-10:40	AM-3014	FEM Simulation on Nanofluid Flow over Power Law Stretching Sheet with MHD Thermo-Diffusive Effect	Rama Bhargava

<b>Session C1: Numerical Analysis</b> <b>Chair: Vedat Suat Erturk</b> <b>Room:SBA0005</b>			
<b>Time</b>	<b>ID</b>	<b>Title</b>	<b>Speaker</b>
10:30-10:55	NA-3005	The Application of Differential Transform Method to a BVP arising in Chemical Reactor Theory	Vedat Suat Erturk
10:55-11:20	AM-3009	A Third-Order Shear Deformation Theory for Free Vibration Analysis of Functionally Graded Shells	Mohammad Zannon
11:20-11:45	CM-3003	Usage of the Randomized Kernel Functional Numerical Algorithm	V. Voytishek
11:45-12:10	NA-3003	A Family of Second Derivative Simpson's Type Block Methods for Stiff Systems	Yohanna Awari
12:10-12:35	NA-3007	Fourth Order Numerical Scheme for Two-Dimensional Inhomogeneous Distributed Order Riesz Space-Fractional Diffusion Equation	Muhammad Yousuf

## Instability boundaries of double-diffusive convection in a Brinkman bidisperse porous medium with an anisotropic permeability effect

Sara H. Saleh, Shatha A. Haddad

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☐ Oral presentation

☐ Poster Presentation

### Abstract:

The problem of double-diffusive convection in a bidisperse porous medium saturated with an incompressible Newtonian fluid is studied. The Brinkman model is employed for the momentum equation. We concentrate on the case where the saturated layer is heated below-salted above and heated below-salted below. Linear stability theory is performed to derive a Rayleigh number threshold for stationary and oscillatory modes. The effect of Brinkman term, anisotropy parameter, and salt Rayleigh number on the onset of convection is discussed graphically. It is found that the presence of Brinkman term with anisotropy parameter has pronounced effect on the stability of the system.

**Keywords:** double-diffusive , bidisperse porous medium, anisotropy.

**2010 Mathematics Subject Classification:**

## THE BEST KNOWN INTERIOR POINT ALGORITHM FOR LINEAR OPTIMIZATION PROBLEM

El Amir Djeflal

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### Abstract:

In this paper, we propose a primal-dual interior-point algorithm for linear optimization (LO) based on a class of kernel functions which is eligible. New search directions and proximity measures are defined based on these functions. We show that the algorithm has  $O(pn^{\frac{(p+1)}{(2p)}} \log(n/\epsilon))$  and  $O(pn^{\frac{1}{2}} \log(n/\epsilon))$ ,  $p > 0$ , complexity results for small and large-update methods, respectively. For its numerical tests some strategies are used and indicate that the algorithm is efficient.

**Keywords:** Interior point method; Optimization; Kernel function.

**2010 Mathematics Subject Classification:** 90C30, 90C51

### References

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## A DISCONTINUOUS GALERKIN METHOD FOR SYSTEMS OF STOCHASTIC DIFFERENTIAL EQUATIONS WITH APPLICATIONS TO POPULATION BIOLOGY, FINANCE, AND PHYSICS

**Helmi Temimi<sup>#</sup>, Mahboub Baccouch<sup>‡</sup>, Mohamed Ben-Romdhane<sup>\$</sup>**

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<sup>‡</sup> Department of Mathematics, University of Nebraska at Omaha, Omaha, NE 68182, USA, Email: *mbaccouch@unomaha.edu*.

### **Abstract:**

In this study, we propose a discontinuous Galerkin (DG) method for systems of stochastic differential equations (SDEs) driven by  $m$ -dimensional Brownian motion. We first construct a new approximate system of SDEs on each element using whose converges to the solution of the original system. The new system is then discretized using the standard DG method for deterministic ordinary differential equations (ODEs). We prove that the proposed scheme is convergent in the mean-square sense. Several linear and nonlinear test problems are presented to show the accuracy and effectiveness of the proposed method. In particular, the proposed scheme is illustrated by considering different examples arising in population biology, physics, and mathematical finance.

**Keywords:** Stochastic differential equation; discontinuous Galerkin method; Wong-Zakai approximation.

**2010 Mathematics Subject Classification:** 65C20, 65C30, 65L20, 65L60, 60H10, 60H20.

FEM SIMULATION ON NANOFLUID FLOW OVER POWER LAW STRETCHING SHEET  
WITH MHD THERMO-DIFFUSIVE EFFECT

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**Abstract:**

The present paper uses the Galerkin Finite Element Method to study numerically the triple diffusive boundary layer flow of homogenous nanofluid over power-law stretching sheet under the effect of external magnetic field. The fluid is composed of nanoparticles along with dissolved solutal particles in the base fluid. The chief mechanisms responsible for enhancement of convective transport phenomenon in nanofluids-Brownian Motion, Diffusiophoresis and Thermophoresis have been considered. The simulations performed in this study are based on the boundary layer approach. With the Heat flux and nanoparticle mass flux boundary conditions, heat transfer, solutal and nanoparticle mass transfer are investigated for different values of controlling parameters i.e. Brownian motion parameter, thermophoresis parameter, magnetic parameter and stretching parameter.

It is observed that the external magnetic field enhances the thermal and solutal boundary layer, thus can be used to control the heat transfer, specially in case of smart cooling devices. Further strengthening the values of Brownian motion and thermophoresis effects for nanoparticles, the heat transfer rate is enhanced. This idea is applicable for industrial process like extrusion of metal sheets, manufacturing of tetrapacks etc. The stretching parameter is found to have a considerable effect in heat and mass transfer rates, a phenomena used for the plastic manufacturing. The result obtained are in good agreement with the earlier result for specialized value and code is also validated. All the results are shown graphically.

**Keywords:** Nanofluid, FEM

**2010 Mathematics Subject Classification:** 76, 65N35

**References:**

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## THE APPLICATION OF DIFFERENTIAL TRANSFORM METHOD TO A BVP ARISING IN CHEMICAL REACTOR THEORY

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### **Abstract:**

In this study, we deal with the numerical solution of the mathematical model for an adiabatic tubular chemical reactor which processes an irreversible exothermic chemical reaction. For steady state solutions, the model can be reduced to the following nonlinear ordinary differential equation [1]:

$$u'' - \lambda u' + \lambda \mu (\beta - u) \exp(u) = 0, \quad (1)$$

where  $\lambda, \mu$  and  $\beta$  are Péclet number, Damköhler number and adiabatic temperature rise, respectively.

Boundary conditions of Eq. (1) are

$$u'(0) = \lambda u(0), u'(1) = 0. \quad (2)$$

Differential transform method [2] is used to solve the problem (1)-(2) for some values of the considered parameters. Residual error computation is adopted to confirm the accuracy of the results. In addition, the obtained results are compared with those obtained by other existing numerical approach [3].

**Keywords:** Adiabatic tubular chemical reactor; Boundary value problems; Differential transform method

**2010 Mathematics Subject Classification:** 34B15, 34K10, 65L10

### **References**

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## A Third-order shear deformation theory for free vibration analysis of functionally graded Shells

**Mohammad Zannon**

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### **Abstract:**

Free vibration problems of functionally graded shells will be consider in this paper, the analysis will be perform by collecting the radial basis functions, according to Third shear deformation theory that accounts for through the thickness deformation using the principle of virtual work to reset Carrera's Unified Formulation with further interpolated by collocation with radial basis functions we will obtain the equations of motion and the boundary conditions. Numerical results will include spherical shell panels with all edges clamped or simply supported and demonstrate the accuracy of the present approach.

**Keywords:** Spherical shells, free vibration, functionally graded materials, Carrera's Unified Formulation.

## USAGE OF THE RANDOMIZED KERNEL FUNCTIONAL NUMERICAL ALGORITHM

T. E. Bulgakova, N.V.Tracheva, **A. V. Voytishek**

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### Abstract:

In the papers [1–3], it was supposed that the randomized kernel functional numerical algorithm can be more effective (to compare with the mesh and the projection functional algorithms) for numerical approximation of solutions of practically important Fredholm integral equations of the second kind. Nevertheless, our calculations for the well-known test one-dimensional integral equation (see, for example, [4]) show that the kernel algorithm is not the most effective. The analogous results also exist in the dissertations [5, 6], where the comparison of kernel and mesh functional algorithms was provided. The most informative applications of the kernel algorithm are presented in the papers [7–10]. We also tried to use this algorithm for estimating angular distributions of polarized radiation. However, the results are worse than for the projection algorithm [11].

**Keywords:** randomized kernel functional numerical algorithm; testing; applications

**2010 Mathematics Subject Classification:** 65C05

### References

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## A Family of Second Derivative Simpson's Type Block Methods for Stiff Systems

**Yohanna Sani Awari**

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Kumleng Micah Geoffrey

: Department of Mathematics, University of Jos, Jos. E-mail: [gkumleng@gmail.com](mailto:gkumleng@gmail.com)



Oral presentation



### **Abstract:**

We describe a new family of self starting second derivative Simpson's type block methods of uniform order  $p = 2k + 2$  for step number  $k \leq 6$  for the solution of ordinary differential equation. The new block methods for  $k = 2, 3, \dots, 6$  were seen to possess good stability property as they were found to be consistent and zero stable. The methods were also shown to be A-stable hence suitable for the numerical integration of stiff systems of ordinary differential equations. Some numerical examples were considered and results obtained show improved accuracy in terms of their maximum absolute errors when compared with the work of existing scholars. Analysis of the solution curves shows that the new block methods approximates well with a stiff Ode Solver (Ode23s).

**Keywords:** Second Derivative LMM, Stiff Differential Equation, Ode Solver, Simpson's Method.

## FORTH ORDER NUMERICAL SCHEME FOR TWO-DIMENSIONAL INHOMOGENEOUS DISTRIBUTED ORDER RIESZ SPACE-FRACTIONAL DIFFUSION EQUATION

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### Abstract:

Distributed-order Riesz space-fractional diffusion equations are used to describe physical processes that lack power-law scaling. A fourth order numerical scheme is developed for a class of initial-boundary value problems associated with these equations. The scheme is based on using Gaussian quadrature rule for the distributed-order Riesz space derivative and Padè approximations of the matrix exponential function. A computationally efficient parallel version of the method is developed using partial fraction splitting technique. Convergence of the scheme is proved analytically and demonstrated through numerical experiments.

**Keywords:** Distributed-order; Riesz space-fractional diffusion; Padè approximation

**2010 Mathematics Subject Classification:** 97N40, 97N50, 97N60, 97N80

### References

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# Advanced Numerical Methods and their Applications

*Special Session Organized by Youssef Belhamadia*

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Special Session A2: Advanced Numerical Methods and their Applications			
Chair: Ali Sayfy			
Room: SBA0005			
Time	ID	Title	Speaker
16:00-16:25	CM-3002	An Efficient approach for the solution of fractional BVPs	Ali Sayfy
16:25-16:50	CM-3006	Multilevel Iteration for the Nonlinear Mild-Slope Equation	Yogi A. Erlangga
16:50-17:15	CM-3001	Numerical Solution of Stochastic Partial Differential Systems with Additive Noise on Overlapping Subdomains	Mostafa Zahri
17:15-17:40	AM-3013	Numerical Determination of an Optimal Control for a Population Dynamics Model	M. Alahyane
17:40-18:05	CM-3004	Modeling and Dynamic Analysis of Two Weakly-Coupled Microbeams under Electrostatic Actuation	Muhammad Alkaddour
18:05-18:30	CM-3007	Dynamics of Metamaterial Beam Equipped with Vibration Absorbers	Ehab Basta

## An efficient approach for the solution of fractional BVPs

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†Professor, Department of Mathematics and Statistics, AUS, UAE, [skhoury@aus.ed](mailto:skhoury@aus.ed)

‡Professor Emeritus, Department of Mathematics and Statistics, AUS, UAE, [sayfy@aus.edu](mailto:sayfy@aus.edu)

### Abstract:

The aim of this study is to present an alternative approach for the numerical solution of a class of fractional boundary value problems (FBVPs). The method is based on first constructing an integral operator that is expressed in terms of the Green's function corresponding to the linear differential term in the fractional differential equation (FDE). Then, fixed point iterative procedures, such as Picard's and Mann's, are applied to the operator to generate an iterative scheme that yields a convergent semi-analytical solution. Numerical examples are reported to confirm the efficiency, reliability, accuracy and fast convergence of the scheme. abstract is not to exceed one page.

**Keywords:** Fractional order BVPs; Green's function; Fixed point iterative schemes..

**2010 Mathematics Subject Classification:** 26A33; 34A12

## MULTILEVEL ITERATION FOR THE NONLINEAR MILD-SLOPE EQUATION

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### Abstract:

Ocean waves approaching coastal areas interact with natural boundaries – irregular seabed, vegetation, e.g., - and man-made structures, transforming the train wave structures into shoaling, wave breaking, and turbulence, to name a few. Modelling such an interaction fundamentally requires full 3D approach. This approach may however be computationally prohibitive if the domain of interests corresponds to an extremely large physical area of order of hundreds of kilometers. A viable computational solution method can be devised by first simplifying the problem via dimensional reduction, reducing a 3D problem to a 2D problem, associated with surface waves, leading to the so-called mild-slope equation:

$$\nabla_h \cdot (u_0 \nabla \phi) + K^2(\phi)\phi = 0, \quad (1)$$

where  $\phi = \phi(x, y)$  is the velocity potential,  $u_0 = \frac{n}{k} \tanh(kh)$ , and  $K$  a modified wavenumber that depends on the solution  $\phi$  and recaptures complicated physical modeling aspects of the waves, which have lost from the original mathematical model (based on the Navier-Stokes equations, e.g.)

In this talk, we shall present a numerical method to solve the mild-slope equation in the transformed form, which corresponds to the generalized nonlinear Helmholtz equation. Since the problem size is typically large, we shall focus on the iterative methods, based on fixed-point method for the non-linear solves and a multilevel iteration for the linear solves. Convergence and numerical results will be presented for model problems.

**Keywords:** Mild-slope, Krylov method, multilevel method,

**2010 Mathematics Subject Classification:** 65F10, 65N06, 65N22, 76B15 (Special session on : Advanced Numerical Methods and their Application)

## NUMERICAL SOLUTION OF STOCHASTIC PARTIAL DIFFERENTIAL SYSTEMS WITH ADDITIVE NOISE ON OVERLAPPING SUBDOMAINS

**Mostafa Zahri**

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### **Abstract:**

In this paper, we present a new numerical approach for solving a class of stochastic partial differential systems (SPDSs) with additive noise on overlapping subdomains. We combine the domain decomposition algorithm (DDA), the deterministic method of lines (MOL) and the barycentric interpolation method (BIM). Together with the DDA-MOL-BIM procedure, we implement the stochastic Ito-Taylor scheme (SIT) for solving a stochastic advection-diffusion-reaction problem. The solution of the SPDSs is then carried out by collecting interior and interface solutions. Finally, computational results are performed on two dimensional overlapped subdomains with nonlinear boundaries.

**Keywords:** Domain-decomposition, barycentric interpolation, stochastic advection-diffusion

**2010 Mathematics Subject Classification:** 35K57, 35R60, 60H15, 60H35

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## Numerical determination of an optimal control for a population dynamics model

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### Abstract:

In this work, we studied the numerical approach of an inverse problem, which related to the source term of a population dynamics model. Therefore, the problem is formulated as a constrained minimization problem and the numerical approach is based on the augmented Lagrangian method. As an application, we will look for a relevant (optimal) controls acting on a small sub-domain of (0,1) steering the studied population to extinction (i.e, the null controllability problem) or at least the approximate null controllability problem in a finite control time.

**Keywords:** Population dynamics, Optimal control, Optimization, Lagrangian method.

**2010 Mathematics Subject Classification:**

**Special session on :** Advanced Numerical Methods and their Applications

## References

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## MODELING AND DYNAMIC ANALYSIS OF TWO WEAKLY-COUPLED MICROBEAMS UNDER ELECTROSTATIC ACTUATION

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### Abstract:

In this work, we conduct a numerical investigation of the dynamic behavior of two weakly coupled Euler-Bernoulli cantilever microbeams subject to electrostatic actuation. The microsystem comprises two cantilever beams with different lengths (in the micro-scale) which are mechanically coupled by a third beam. This design has been proposed as a MEMS resonator sensor. We first consider the linearized system obtained by expanding the nonlinear electrostatic forcing term into Taylor series and apply the single-mode Galerkin discretization approach to simulate the response of the resonator. The numerical results are compared against existing experimental data in the literature [1]. A good agreement between the two sets of data is obtained. We vary the DC voltage and compute the natural frequencies and amplitude ratios in this regime. This is performed when considering low actuating voltage, away from the pull-in instability where nonlinear effects are expected to be more prominent. We then investigate the behavior of the full nonlinear system by studying the beam vibrations using the multimode Galerkin discretization and incorporate the nonlinear term of the electrostatic force to gain a better understanding of the system dynamics. We first validate the convergence of the numerical solution to the multimode discretization problem when varying the number of expansion modes, since it is expected that a solution with a sufficient finite number of modes would approximate the actual solution within reasonable accuracy. While the linearized model shows good predictive capability of the dynamic response of the resonator when applying low voltages, the nonlinear model enables to capture the system dynamics when operating at higher voltages, near the pull-in instability as needed for several MEMS applications such as switching.

**Keywords:** MEMS resonators, nonlinear dynamics, Galerkin method

**2010 Mathematics Subject Classification:** Special session on: Advanced Numerical Methods and their Applications

### References

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## DYNAMICS OF METAMATERIAL BEAM EQUIPPED WITH VIBRATION ABSORBERS

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### Abstract:

Vibration control has been a subject of constant research interest for the past years. Among the numerous designs that have been proposed and implemented to allow vibration mitigation, is the concept of metastructures or mechanical metamaterials [1]. Metastructures, in general, are structures equipped with distributed periodic vibration resonators (absorbers). In the present study, we analyze the effect of nonlinearities on a simply supported Euler-Bernoulli beam (host) embedded with a periodic arrangement of spring-mass-damper subsystems deployed for vibration absorption. Moreover, the mass is conserved which means that any added mass in the resonators is taken from the host structure itself. Our main goal is to show that the vibration mitigation can be greatly improved by optimally tuning the frequencies of the periodic subunits in the metastructures. We develop the mathematical model of the metastructure to perform the linear free and forced vibrations analyses based on the Euler-Bernoulli model [2]. The mathematical model is a set of integro-partial differential equations governing the coupled vibrations of the beam and absorbers. These equations are projected onto the model space and expressed in matrix form by using the Galerkin discretization approach. The linear regime analysis helps in gaining insight on the effect of the local resonators on the mitigation of the oscillations of the host beam. The heart of the metastructures concept shows up by their intriguing ability to suppress several natural frequencies simultaneously. Furthermore, the study is extended to explore the nonlinear dynamic aspects of the vibrating system. State space vectors are obtained to handle the set of nonlinear ODEs. The numerical study reveals that the amplitude of the host beam, when being excited in the vicinity of the natural frequencies, can be greatly mitigated by the proper tuning of the local absorbers. This study demonstrates the ability of the metastructure to withstand external loadings even when operating near resonance. Finally, the optimizer algorithm tool in Matlab (pattern search) is integrated with the nonlinear mathematical model to identify the number of absorbers needed along with their tuning frequencies to maximize the vibration suppression.

**Keywords:** Mechanical metamaterial, Nonlinear vibration, Vibration suppression

**2010 Mathematics Subject Classification:** "Special session on : Advanced Numerical Methods and their Applications"

### References

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# Partial Differential Equations, Analysis and Control

*Special Session Organized by Amjad Tuffaha*

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<b>Special Session A2: Partial Differential Equations, Analysis and Control</b> <b>Chair: Amjad Tuffaha</b> <b>Room:SBA0012</b>			
Time	ID	Title	Speaker
16:00-16:25	AM-3010	Some Aspects of Kuramoto-Sivashinsky Equations	Said Benachour
16:25-16:50	AM-3004	Evolution of Elementary Waves in Two Phase Mass Flows	Manoj Kumar Pandey
16:50-17:15	DE-3012	Analysis of Burgers- $\alpha$ Equation: Optimal Estimates of Parameter $\alpha$ using Physics-informed Deep Learning Algorithm	Bong-Sik Kim
17:15-17:40	AN-3019	Lower Semi-continuity and Convergence of a Class of Linear Growth Functionals with L1 Data	Thomas Wunderli
17:40-18:05	DE-3003	Global Existence and Stability for Coupled System of Hyperbolic Equations with Variable Exponents	Oulia Bouhoufani
<b>Special B1: Partial Differential Equations, Analysis and Control</b> <b>Chair: Abdelaziz Soufyane</b> <b>Room:SBA0012</b>			
Time	ID	Title	Speaker
9:00-9:50	DE-3001	A Stability Result for a Nonlinear Damped Wave Equation with Variable-Exponent Nonlinearities	Salim Messoudi
9:50-10:15	AN-3017	Translation Operator and Maximal Function for the (K, 1)-Generalized Fourier Transform	Salem Ben Said
10:15-10:40	FM-3001	Numerical Solution of an Integral Equation for Perpetual Bermudan Options	Ghada Alobaidi
<b>Special Session B2: Partial Differential Equations, Analysis and Control</b> <b>Chair: Amjad Tuffaha</b> <b>Room:SBA0012</b>			
Time	ID	Title	Speaker
11:00-11:50	AM-3031	On some Inverse Boundary Value Problems related to the Monodomain Model of Cardiac Electrophysiology	Elena Beretta
<b>Special Session B3: Partial Differential Equations, Analysis and Control</b> <b>Chair: Ghada Alobaidi</b> <b>Room:SBA0012</b>			
Time	ID	Title	Speaker
14:00-14:50	DE-3022	Loss of Regularity for Transport Equations and Optimal Mixing	Anna Mazzucato
14:50-15:15	AN-3013	Some Questions related to Optimality of the Energy Behaviour of Reissner-Mindlin-Timoshenko Systems	Makram Hamouda

15:15-15:40	DE-3007	Global Attractors for Quasilinear Parabolic-Hyperbolic Equations Governing Longitudinal Motions of Nonlinearly Viscoelastic Rods	Suleyman Ulusoy
15:40-16:05	DE-3014	Eigenvalues of the Third Boundary Problem for Bitsadze Equation	Alip Mohamed
<b>Session C1: Partial Differential Equations, Analysis and Control</b> <b>Chair: Amjad Tuffaha</b> <b>Room:SBA0012</b>			
Time	ID	Title	Speaker
10:30-11:20	DE-3023	TBA	Nader Masmoudi
11:20-11:45	DE-3009	Memory-Type Boundary Control of a Laminated Timoshenko Beam	Abdelaziz Soufyane
11:45-12:10	AN-3017	A Convexity Problem for a Semi-Linear PDE	Layan Elhajj
<b>Special Session C2: Partial Differential Equations, Analysis and Control</b> <b>Chair: Cristian Enache</b> <b>Room:SBA0012</b>			
Time	ID	Title	Speaker
14:00-14:25	DE-3005	Determining Functionals for a Non-Conservative, Nonlinear Plate Equation	Justin Webster
14:25-14:50	DE-3016	Global Well-Posedness of the Cauchy Problem for the Jordan-Moore-Gibson-Thompson Equation	Belkacem Said-Houari
14:50-15:15	AN-3010	An Introduction to Metrics and their Uses in Complex Analysis	Ziyad Adwan
15:15-15:40	DE-3015	A Monotonicity Property of the $\mathbb{S}^n$ -Torsional Rigidity	Cristian Enache

## SOME ASPECTS OF KURAMOTO-SIVASHINSKY EQUATIONS

Said Benachour

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**Abstract:** In a smooth and bounded domain  $\Omega$  of  $n$  spacial dimensions, the Kuramoto–Sivashinsky equation:

$$\phi_t + \Delta^2 \phi + \Delta \phi + \frac{1}{2} |\nabla \phi|^2 = 0, \quad \text{in } \Omega \times (0, T). \quad (1)$$

subject to the appropriate initial and boundary conditions, is an amplitude equation that appears in hydrodynamics and in combustion theory as a model for the propagation of flame fronts. To avoid dealing with the average of the solution to (1) most authors consider instead the system of equations for  $U = \nabla \phi$ :

$$U_t + \Delta^2 U + \Delta U + (U \cdot \nabla) U = 0, \quad \text{in } \Omega \times (0, T). \quad (2)$$

in which the nonlinearity takes a more familiar advection form and which is called also the Kuramoto–Sivashinsky equation.

In the one-dimensional case, the equations (1) and (2) was studied by several authors both analytically and computationally. The Cauchy problem is well posed : there exists a unique solution, continuously dependent on the initial data in suitable spaces.

The question of the global well-posedness for the two and three dimensional Kuramoto–Sivashinsky equations is one of the open question in nonlinear analysis. The main obstacle in this challenging problem is the lack of a maximum principle because the presence of the fourth order term  $\Delta^2$ . In order to understand the nature of the evolution, in the Kuramoto–Sivashinsky equation, it is convenient to consider the mechanisms involved by the linear part, the nonlinear part and their somewhat hidden competitions. Indeed, the growing modes in the linear part depends upon the size of  $\Omega$ .

In order to illustrate these problems, we shall introduce and discuss the hyper-viscous Hamilton–Jacobi-like equations, parameterized by the exponent  $p \geq 1$ , for the scalar function  $\psi$ :

$$\psi_t + \Delta^2 \psi = |\nabla \psi|^p \quad \text{in } \Omega \times (0, T). \quad (3)$$

In the talk, we will try to give a grasp of difficulties arising in considering that kind of equations and give some positive answers to above questions.

**Keywords:** Kuramoto–Sivashinsky equations.

**35G30; 35J40; 35K25:**

## References

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## EVOLUTIN OF ELEMENTARY WAVES IN TWO PHASE MASS FLOWS

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### Abstract:

Lie's method, which is based on mapping a system of partial differential equations (PDEs) to an equivalent system of PDEs is one of the most efficient analytical method to find exact particular solution of linear and nonlinear PDEs (see, Bluman and Anco [1] and Olver [2]). Exact particular solutions of the nonlinear system of PDEs plays important role to provide useful information towards our understanding of the complex physical phenomenon involved. Special exact solutions of the system of nonlinear PDEs are of great interest; these solutions play a major role in designing, analyzing and testing numerical methods for solving special initial and/or boundary-value problems. Many researchers have studied nonlinear systems of PDEs using the classical and non-classical symmetries admitted by the system, these symmetries are then used to obtain a large class of similarity solutions to the governing PDE.

In this paper, we apply the Lie's method to the two-phase drift flux model (see, [3] and [4]) and obtained the infinitesimal symmetries of the governing system of non-linear PDEs, the Lie algebra in the present case is infinite dimensional. An exact particular solution of the two-phase model is obtained by mapping the governing system to an equivalent system of PDEs, where a constant solution of the reduced PDEs is mapped to a non-constant solution of the two-phase model. The special solution which exhibits space time dependence is used to study the evolutionary behaviour of the contact and weak discontinuity waves. The transport equations for the weak wave and the characteristic shock are obtained explicitly within the state characterized by the special solution and certain interesting observations are made.

**Keywords:** Lie group of transformations; Weak discontinuity ; Contact discontinuity.

**2010 Mathematics Subject Classification:** 70G65, 35L60, 35A30

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## Analysis of Burgers- $\alpha$ Equation: Optimal Estimates of Parameter $\alpha$ Using Physics-informed Deep Learning Algorithm

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**Abstract:** The Burgers- $\alpha$  equation (or called the Leray-regularized Burgers equation) is a Hamiltonian regularization of the Burgers equation. It is a quasilinear evolution equation that consists of the inviscid Burgers equation plus  $\mathcal{O}(\alpha^2)$  nonlinear terms:

$$u_t + uu_x - \alpha^2 u_{txx} - \alpha^2 uu_{xxx} = 0.$$

For smooth initial data  $u(x, 0)$  which decreases at least one point (so there exists  $y$  such that  $u_x(y, 0) < 0$ ), the classical solution  $u(x, t)$  of the inviscid Burgers equation (when  $\alpha = 0$ ) fails to exist beyond a specific finite break time  $T > 0$ . It is because the characteristics of the inviscid equation cross in a finite time. The Burgers- $\alpha$  equation bends its characteristics slightly out of the way of one another, avoiding any finite-time intersection and remedy the finite-time breakdown. So, the Burgers- $\alpha$  equation possesses a classical solution globally in time for smooth initial data for  $\alpha > 0$ . Furthermore, the solution  $u^\alpha(x, t)$  of the Burgers- $\alpha$  equation converges strongly, as  $\alpha \rightarrow 0$ , to the *unique entropy solution* of the Cauchy problem for the inviscid Burgers equation. One of the open questions is how to understand and choose the length scale  $\alpha$ . The size  $\alpha$  is the length scale below which the smaller physical phenomena are averaged out. In its numerical interpretation, one practical rule of thumb has often been chosen to  $\alpha$  as some small integer multiple of the minimum grid spacing. Here we use the *physics-informed neural network* to estimate the optimal value of  $\alpha$  closely approximating the exact Burgers equation and simulate the learned new Burgers- $\alpha$  equation to compare with the original Burgers equation. We will also demonstrate that it correctly captures physical shocks by solving the Riemann problem.

**Keywords:** Burgers equation, Physics-informed Neural Network

**2010 Mathematics Subject Classification:** 35, 37, 65

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## LOWER SEMICONTINUITY AND $\Gamma$ -CONVERGENCE OF A CLASS OF LINEAR GROWTH FUNCTIONALS WITH $L^1$ DATA

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**Abstract:** We prove some new results for linear growth functionals  $\int_{\Omega} \varphi(x, Du)$ ,  $u \in BV(\Omega)$ , where

$$\varphi(x, p) = \begin{cases} g(x, |p|) & \text{if } |p| \leq \beta \\ \psi(x)|p| + k(x) & \text{if } |p| > \beta, \end{cases}$$

$\psi \in C(\Omega) \cap L^{\infty}(\Omega)$ ,  $\psi \geq 0$  on  $\Omega$ , and  $\varphi$  is convex in  $p$ . In particular, we give conditions on  $\varphi$  for which  $\int_{\Omega} \varphi(x, Du)$  is lower semicontinuous in  $L^1(\Omega)$  with  $k \in L^1(\Omega)$  and  $g(\cdot, |p|) \in L^1(\Omega)$  for each  $p \in \mathbb{R}^N$ . Notably, we make no continuity or lower semicontinuity assumptions for  $\varphi$  in  $(x, p)$  and no differentiability assumption for  $\varphi$  in  $p$ , as is done in earlier work. We also consider more general linear growth functionals  $\int_{\Omega} g(x, |Du|)$  with  $g(x, |p|)$  convex in  $|p|$  and prove  $\Gamma$ -convergence of functionals of the form  $\int_{\Omega} \varphi(x, Du)$  to  $\int_{\Omega} g(x, |Du|)$ . Finally, functionals with specified trace values for  $u$  are also considered.

**Keywords:** linear growth, lower semicontinuity, bounded variation

**2010 Mathematics Subject Classification:** 49N

GLOBAL EXISTENCE AND STABILITY FOR COUPLED SYSTEM OF HYPERBOLIC EQUATIONS WITH VARIABLE EXPONENTS

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**Abstract:**

In this work, we consider a coupled system of two nonlinear hyperbolic equations with variable exponents in the damping and source terms. Under a suitable assumptions on the initial data, we prove the global existence and we establish the stability result for the solutions.

**Keywords:** Global existence, Hyperbolic equation, Stability.

**2010 Mathematics Subject Classification:** 35L05, 35B40, 35L70, 93D20.

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## A stability result for a nonlinear damped wave equation with variable-exponent nonlinearities

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### **Abstract:**

In this work, we consider the following nonlinear wave equation with variable exponents:

$$u_{tt} - \operatorname{div}[|\nabla u|^{r(x)-2}\nabla u] + |u_t|^{m(x)-2}u_t = 0,$$

in a bounded domain. This type of equations arise in modeling many physical phenomena such as flows of electro-rheological fluids or fluids with temperature-dependent viscosity, nonlinear viscoelasticity, filtration processes through a porous media and image processing. By using a lemma by Komornik, we prove the decay estimates for the solution under suitable assumptions on the variable exponents  $m, r$  and the initial data.

## Translation operator and maximal function for the $(k, 1)$ -generalized Fourier transform

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**Abstract:** In this talk I will report on a recent work with L. Deleaval where we study a translation operator associated with the  $n$ -dimensional  $(k, 1)$ -generalized Fourier transform [?], where  $k$  is a multiplicity function for the Dunkl operators. A crucial result shows that this operator is a positivity-preserving operator on a suitable space of radial functions on  $\mathbb{R}^n$ . As an application, a Hardy-Littlewood type maximal operator was defined, and a weak-type  $(1, 1)$  and a strong type  $(p, p)$  estimates for this maximal operator with a precise behavior in  $n$  and  $k$  are established.

**Keywords:** Generalized Fourier transform; generalized translation operator; Hardy-Littlewood type maximal operator.

**2010 Mathematics Subject Classification:** Primary 42B25

## References

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## NUMERICAL SOLUTION OF AN INTEGRAL EQUATION FOR PERPETUAL BERMUDAN OPTIONS

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### **Abstract:**

We consider perpetual Bermudan options, which have no expiration and can be exercised every  $T$  time units. We use the Green's function approach to write down an integral equation for the value of a perpetual Bermudan call option on an expiration date; this integral equation leads to a Wiener-Hopf problem. We discretize the integral in the integral equation to convert the problem to a linear algebra problem, which is straightforward to solve, and this enables us to find the location of the free boundary and the value of the perpetual Bermudan call. Finally we compare our results to earlier studies which used other numerical methods.

**Keywords:** Bermudan options; integral equation; Wiener-Hopf problems.

**2010 Mathematics Subject Classification:** 91G20

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## ON SOME INVERSE BOUNDARY VALUE PROBLEMS RELATED TO THE MONODOMAIN MODEL OF CARDAC ELECTROPHYSIOLOGY

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☐ Oral presentation ☐

### Abstract:

Ischemic heart disease results from a restriction in blood supply to the heart and represents the most widespread heart disease. Detecting ischemic regions at early stages of their development from noninvasive (or minimally invasive) measurements is thus of primary importance. This is usually performed by recording the electrical activity of the heart, by means of either body surface or intracardiac measurements. Mathematical and numerical models of the cardiac electrophysiology can be used to shed light on the potentialities of electrical measurements in detecting ischemias. More specifically, the goal is to combine boundary measurements of (body-surface or intracavitary) potentials and a mathematical description of the electrical activity of the heart in order to identify the position, the shape and the size of heart ischemias and/or infarctions. The cardiac electrical activity can be comprehensively described in terms of the monodomain model, consisting of a boundary value problem for a semilinear reaction-diffusion equation coupled with nonlinear ordinary differential equations. In my talk, I will analyze the case of an insulated heart neglecting the coupling with the torso. This results in the challenging inverse problem of detecting conductivity inclusions for the monodomain system with a single measurement of the endocardial potential. I will start first analyzing the steady-state version of the monodomain model since it already exhibits the main features and difficulties of the time-dependent model. Then, I will go through some recent results obtained in the time dependent case.

**Keywords:** inverse problems, nonlinear reaction diffusion systems, electrophysiology

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## Loss of regularity for transport equations and optimal mixing

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**Abstract:** I will discuss examples of optimal mixers in incompressible flows and how these examples provide a complete loss of regularity result for the continuity/transport equation. This is joint work with Giovanni Alberti (Pisa) and Gianluca Crippa (Basel).

**Keywords:** Optimal Mixing

## SOME QUESTIONS RELATED TO OPTIMALITY OF THE ENERGY BEHAVIOUR OF REISSNER-MINDLIN-TIMOSHENKO SYSTEMS

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### Abstract:

In this talk, we are dealing with a 2D Reissner-Mindlin Timoshenko model subject to a nonlinear dissipation acting on the equations of the rotation angles. The set of equations for this 2D system reads as follows:

$$\rho_1 w_{tt} - K(\psi + w_x)_x - K(\varphi + w_y)_y = 0, \text{ in } \Omega \times \mathbb{R}^+, \quad (1)$$

$$\rho_2 \psi_{tt} - D\psi_{xx} - D\left(\frac{1-\mu}{2}\right)\psi_{yy} - D\left(\frac{1+\mu}{2}\right)\varphi_{xy} + K(\psi + w_x) + \chi_1(\psi_t) = 0, \text{ in } \Omega \times \mathbb{R}^+, \quad (2)$$

$$\rho_2 \varphi_{tt} - D\varphi_{yy} - D\left(\frac{1-\mu}{2}\right)\varphi_{xx} - D\left(\frac{1+\mu}{2}\right)\psi_{xy} + K(\varphi + w_y) + \chi_2(\varphi_t) = 0, \text{ in } \Omega \times \mathbb{R}^+, \quad (3)$$

where  $\Omega \subset \mathbb{R}^2$  is bounded,  $\rho_i$  ( $i = 1, 2$ ) is the (constant) mass per unit of surface area,  $\mu$  is Poisson's ratio ( $0 < \mu < 1/2$ ),  $D = \frac{Eh^3}{12(1-\mu^2)}$  is the modulus of flexural rigidity,  $K = \frac{kEh}{2(1+\mu)}$  is the shear modulus where  $E$  is the Young's modulus,  $h$  is the (uniform) plate thickness and  $k$  is the shear correction. Here  $\chi_i$  ( $i = 1, 2$ ) is the dissipation term.

Some suitable boundary and initial conditions are associated with (1)–(3).

Following our previous one-dimensional study [2, 3], it is natural to ask the following questions:

- Can we generalize the optimality (upper and lower estimates) study [2, 3] of the 1D Timoshenko beam model?
- What are the types of dissipation leading to the stabilization of the 2D system (1)–(3) and how to find the “optimal” dissipation to produce the exponential or non-exponential stability of the energy?
- Is there any dependence of the stability results on the coefficients in (1)–(3)?

We aim here to start at least answering some of these questions and we begin by proving an existence result. Then we show that a quasi-optimal upper energy decay rate can be obtained to our plate model. The proofs of our results are based on multiplier techniques, weighted nonlinear integral inequalities and the optimal-weight convexity method of [1].

**Keywords:** Optimality, Reissner-Mindlin Timoshenko system, Strong asymptotic stability.

**2010 Mathematics Subject Classification:** 35B35, 35B40, 35L51, 93D20

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## GLOBAL ATTRACTORS FOR QUASILINEAR PARABOLIC-HYPERBOLIC EQUATIONS GOVERNING LONGITUDINAL MOTIONS OF NONLINEARLY VISCOELASTIC RODS

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**Abstract:** We prove the existence of a global attractor and estimate its dimension for a general family of third-order quasilinear parabolic-hyperbolic equations governing the longitudinal motion of nonlinearly viscoelastic rods carrying an end mass and subject to interesting body forces. The simplest version of the equations has the form  $w_{tt} = n(w_x, w_{xt})_x$  where  $n$  is defined on  $(0, \infty) \times \mathbb{R}$  and is a strictly increasing function of each of its arguments, with  $n \rightarrow -\infty$  as its first argument goes to 0. This limit characterizes a total compression, a source of technical difficulty, which new delicate a priori estimates prevent. We determine how the dimension of the attractor varies with the ratio of the mass of the rod to that of the end mass, giving conditions ensuring that the dimension is small. The estimates of dimension illuminate asymptotic analyses of the governing equation as this mass ratio goes to 0. We'll also introduce some recent results on the subject.

**Keywords:** Nonlinearly viscoelastic rods, Dimension of attractors, Quasilinear parabolic-hyperbolic equations.

**2010 Mathematics Subject Classification:** 35B40, 35B41, 70E17

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## EIGENVALUES OF THE THIRD BOUNDARY PROBLEM FOR BITSADZE EQUATION

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**General area of research:** ..... (Partial Differential Equations: Analysis and Control.)

**Abstract:** The eigenvalue problem of the third boundary condition is studied for the Bitsadze equation on the unit disc. It turns out that in general the problem cannot have unique solution unless some additional boundary conditions are imposed. Furthermore, the problem is solvable only if some compatibility conditions are satisfied. Fourier series method are used to obtain the eigenvalues and the corresponding solutions explicitly.

**Keywords:** The third boundary condition, Bitsadze equation, boundary eigenvalues

**2010 Mathematics Subject Classification:**35J25

## MEMORY-TYPE BOUNDARY CONTROL OF A LAMINATED TIMOSHENKO BEAM

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### **Abstract:**

We consider a laminated Timoshenko beam with boundary conditions of a memory-type. This structure is given by two identical uniform layers on top of each other, taking into account that an adhesive of small thickness is bonding the two surfaces and produces an interfacial slip. Under the assumptions of wider classes of kernel functions, we establish an optimal explicit energy decay result. Our result improves earlier results in the literature.

## A CONVEXITY PROBLEM FOR A SEMI-LINEAR PDE

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**Abstract:** In this paper we prove convexity of super-level sets of a semi-linear PDE with a non-monotone right hand side, and with a free boundary

$$\begin{cases} \Delta u = \chi_{\{0 < u < 1\}} & \text{in } \mathbb{R}^n \setminus D, \\ u = 2 & \text{on } \partial D. \end{cases}$$

Here  $D$  is assumed to be convex, and  $n \geq 2$ . The main difficulty of this problem is that the right hand side is non-monotone and no a priori regularity is known about the boundary  $\partial\{u > 0\}$ .

**Keywords:** Convexity, starshapedness, free boundary

**2010 Mathematics Subject Classification:**[2000]35R35

## DETERMINING FUNCTIONALS FOR A NON-CONSERVATIVE, NONLINEAR PLATE EQUATION

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### Abstract:

We establish the existence of a finite set of (asymptotically) determining functionals [5] for dynamics corresponding to a nonlinear plate model arising in aeroelasticity [3]. A panel immersed in an inviscid potential flow is modeled with piston-theoretic aerodynamics appended to a clamped Berger plate in the absence of rotational inertia terms. The effect of the flow provides weak damping as well as a non-conservative lower order term which can destabilize the dynamics (aeroelastic *flutter* [3]). The presence of nonlinearity in the model is essential to provide boundedness of energies in the form of an absorbing ball, in light of the fundamentally non-conservative dynamics [4].

The *quasi-stability* property [1, 2] is shown on the absorbing ball, which yields the existence of a smooth, finite dimensional global attractor [4]. From there, a finite set of asymptotically *determining functionals* is constructed; if two trajectories agree asymptotically in time on such a set of functionals, then they are identical. A set of functionals is shown to be determining by measuring the *completeness defect* via the quasi-stability estimate for some practical functionals on the state space (e.g., nodes, modes, and volume averages). The main result requires *no imposed structural damping*, as requisite dissipative effects are contributed through the aerodynamics.

Apart from the finite fractal dimension of the compact global attractor, the existence of a finite set of determining functions represents an effective *finite dimensionality* for the asymptotic dynamics associated with the aeroelastic system. These two facts can be thought of as a rigorous justification for the treatment of aeroelastic systems through *modal truncation*, as is prevalent in engineering literature [3].

**Keywords:** nonlinear plate, determining modes, quasi-stability.

**2010 Mathematics Subject Classification:** 74F10, 74K20, 35B40, 74B20

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## GLOBAL WELLPOSEDNESS OF THE CAUCHY PROBLEM FOR THE JORDANMOOREGIBSONTHOMPSON EQUATION

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■ Oral presentation

□ Poster Presentation

### Abstract:

We consider the Cauchy problem of a third order in time nonlinear equation known as the Jordan–Moore–Gibson–Thompson (JMGT) equation arising in acoustics as an alternative model to the well-known Kuznetsov equation. First, using the contraction mapping theorem, we show a local existence result in appropriate function spaces. Second, by using the energy method together with a bootstrap argument, we prove a global existence result for small data. Third, polynomial decay rates in time for the solution will be obtained for space dimensions  $N \geq 2$ .

**Keywords:** Global existence, energy method, decay rate.

**2010 Mathematics Subject Classification:** 45N05, 45D05.

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## AN INTRODUCTION TO METRICS AND THEIR USES IN COMPLEX ANALYSIS

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### **Abstract:**

In this talk, we introduce Poincare's metric and its uses and applications in several branches of mathematics; most notably complex analysis and geometry. We will also show Ahlfors' version of Schwartz' Lemma and how it can be used to prove Picard's Theorem. The aim is to find collaborators who would like to join us in research in complex analysis of several variables and behaviour of solutions (and of approximate solutions) of complex vector fields in  $\mathbb{C}^n$ . We will keep prerequisites to a minimum so that undergraduate and graduate students can attend and benefit from this talk.

**Keywords:** Poincare's Metric, Non-Euclidean Geometry, Complex Vector Field.

**2010 Mathematics Subject Classification:** 30A10, 30H05, 30J05, 32H25, 32V10.

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## A MONOTONICITY PROPERTY OF THE $p$ -TORSIONAL RIGIDITY

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□ Poster Presentation

**Abstract:** For a bounded domain  $\Omega \subset \mathbb{R}^N$ ,  $N \geq 2$  and a real number  $p > 1$ , we denote by  $u_p$  the  $p$ -torsion function on  $\Omega$ , that is the solution of the torsional creep problem  $\Delta_p u = -1$  in  $\Omega$ ,  $u = 0$  on  $\partial\Omega$ , where  $\Delta_p := \operatorname{div}(|\nabla u|^{p-2} \nabla u)$  is the  $p$ -Laplace operator. In this talk we are going to present some monotonicity properties for the  $p$ -torsional rigidity on  $\Omega$ , defined as  $T_p(\Omega) := \int_{\Omega} u_p dx$ . More precisely, we first show that there exists  $T \in (0, 1]$  such that for each open, bounded, convex domain  $\Omega \subset \mathbb{R}^N$ , with smooth boundary and  $\delta(\Omega) \leq T$ , where  $\delta(\Omega)$  represent the average integral on  $\Omega$  of the distance function to the boundary of  $\Omega$ , the function  $p \rightarrow T(p; \Omega) := |\Omega|^{p-1} T_p(\Omega)^{1-p}$  is increasing on  $(1, \infty)$ . Moreover, we also show that for any real number  $s > T$ , there exists an open, bounded, convex domain  $\Omega \subset \mathbb{R}^N$ , with smooth boundary and  $\delta(\Omega) = s$ , such that the function  $p \rightarrow T(p; \Omega) := |\Omega|^{p-1} T_p(\Omega)^{1-p}$  is not a monotone function of  $p \in (1, \infty)$ . Finally, we use this result to get a new variational characterization of  $T_p(\Omega)$  in the case when  $\delta(\Omega)$  is small enough.

**Keywords:** distance function to the boundary; torsional rigidity;  $p$ -Laplacian.

**2010 Mathematics Subject Classification:** 35P30; 47J10; 49R05; 49J40; 58C40.

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# Probability Theory and Applications

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Session C2: Probability and Applications Chair: Guillaume Leduc Room:SBA0008			
Time	ID	Title	Speaker
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14:25-14:50	PA-3005	Derivative Formulae for Heat Semigroups on Riemannian Manifolds	James Thompson
14:50-15:15	PA-3006	Decidability of Learning in Finite Settings and Existence of Probabilities	Alberto Gandolfi
15:15-15:40	PA-3007	Convergence of lattice valued options to their Black-Scholes limit	Guillaume Leduc

## HYBRID STOCHASTIC DIFFERENTIAL SYSTEMS IN PHARMACOKINETICS

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### Abstract:

When it comes to drugs, a key factor for the efficacy of any therapy is patient medication compliance. There are several factors influencing compliance including drug type and formulation, disease status, health care system, community care and family. Patient questionnaires allow the identification of some profiles of non-compliers. Here we consider that drug noncompliance is more closely dependent on the patient age and the treatment duration. Since 1986 AARDEX has developed products to measure and analyze patient adherence to prescribed drug dosing regimens in both trials and practice. Patient's drug intakes are electronically monitored by Medication Event Monitoring Systems (MEMS). The MEMS monitors are drug packages with integral electronic micro circuitry designed to compile the dosing histories of ambulatory patients' prescribed medications. Our main objective in this work is to give an advance in the analysis of collected clinical data using tools from applied stochastic analysis.

*Intravenous multi-dosing with full compliance:* The drug is administered in multiple fixed doses  $\{D_i\}_{i=1,2,\dots}$  at some well determined instants  $\{T_i\}_{i=1,2,\dots}$ . Here we assume instantaneous inputs of the drug into the systemic concentration. It is well accepted that kinetics of first order are involved in the elimination process. The following is a single-compartment pharmacokinetic model with elimination rate  $\lambda_e$ :

$$\begin{aligned}\dot{x}(t) &= -\lambda_e x(t) & t \in [T_{i-1}, T_i[ \\ x(T_i) &= x(T_i^-) + D_i\end{aligned}$$

However, some of the regimen features may vary in practice. Here we shall consider random times of drug intake with either deterministic or random doses, along with random and time varying elimination rate. Accordingly, a possible approach towards the study of the drug concentration response  $x(t)$  can only be stochastic. Namely, hybrid stochastic differential systems (HSDS) are the most appropriate choice for modeling irregular or variable compliance. This model induces a Fokker-Planck-Kolmogorov equation along with moment equations, and computations based on direct solutions of the latter make it possible to study the variability of the concentration around its mean as compared to the full compliance case and to assess the effect of some parameters such as the intake and elimination rates. Roughly speaking, a HSDS is a piecewise diffusion process with jumps of two kinds, spontaneous and predictable ones. This model can be applied for intravenous multi-dosing in case of poor patient compliance; the essential tasks for us are to identify all its ingredients and derive its bearings. Of interest are a couple of operators associated to every Markov process: the generating operator and the Fokker-Planck operator. The technical part of this work reviews in the present context classical results of Dynkin's formula and moment equations. These two main results allow one to perform explicit computations in order to obtain precise results describing various aspects of the probability distribution of the concentration that are important for assessing the efficacy of the regimen. Namely, we focus on an aspect of practical relevance: the variability of the drug concentration response.

**Keywords:** compliance, pharmacodynamics, stochastic analysis.

**2010 Mathematics Subject Classification:** 60Hxx, 60J75, 60J25.

### References

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## DERIVATIVE FORMULAE FOR HEAT SEMIGROUPS ON RIEMANNIAN MANIFOLDS

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**General area of research:** Probability Theory and Applications

### Abstract:

A Riemannian manifold is a smooth manifold whose tangent spaces are equipped with a smooth assignment of inner products. The inner products, collectively known as the *metric tensor*, give meaning to length, volume, angle, and other geometric quantities. On each Riemannian manifold there exists an important stochastic process called *Brownian motion*. It is the diffusion process generated by the Laplacian. Brownian particles explore the manifold and many local and global objects can be described in terms of it. For example, the Feynman-Kac formula gives a probabilistic expression for solutions to the heat equation in terms of Brownian motion. The derivatives of these solutions can also be expressed probabilistically, by formulae of the type introduced by Bismut in [1].

In this talk, we suppose  $V$  is a vector field on a smooth manifold and  $P_t$  a semigroup generated by an elliptic diffusion operator. This operator could, for example, be the Laplacian of some Riemannian metric. Using an argument based on local martingales, we prove a probabilistic formula for  $P_t(V(f))$  analogous to Bismut's formula which, as mentioned above, concerns the derivative  $V(P_t f)$ . Just as Bismut's formula can be used to derive Harnack inequalities, our formula can be used to derive *shift-Harnack* inequalities, of the type introduced by Wang in [4]. Our results, building upon those found in [2], have been published in the article [3].

**Keywords:** Brownian Motion, Riemannian Manifold, Ricci Curvature

**2010 Mathematics Subject Classification:** 58J65; 60J60; 53C21

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- [1] Jean-Michel Bismut. *Large deviations and the Malliavin calculus*, volume 45 of *Progress in Mathematics*. Birkhäuser Boston, Inc., Boston, MA, 1984.
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## DECIDABILITY OF LEARNING IN FINITE SETTINGS AND EXISTENCE OF PROBABILITIES

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### Abstract:

It has been recently discovered [1] that machine learnability can be undecidable within the ZFC axioms of Mathematics; the proof shows that even for finitely supported probabilities in  $[0, 1]$ , the existence of a learner for the maximal probability of finite sets is independent of the ZFC axioms.

It becomes thus relevant to explore situations in which learnability is decidable, even if finding the learner might require a prohibitively high number of steps.

To do this, we look at things from the dual point of view of existence of a probability escaping being learned by a potential learner, and show that in a finite setting, learnability is decidable. Machine learning entails the existence of a learning dimension  $n$ : on samples of size  $n$  the learner determines a set of almost maximal probability with high probability. This is actually the most relevant situation, as in practically all applied problems one can easily provide some a-priori bounds to the number of possible states. Hence, the spectre of undecidability does not really appear in all practical contexts.

The proof is done by relating the problem to algebraic geometry and Tarski Seidenberg decidability for first-order theories of real numbers. We then exploit this connection further, showing the existence of a suitable Dutch Book, à la De Finetti, against the believer of an incorrect learning dimension.

**Keywords:** Machine learning, decidability, probability

**2010 Mathematics Subject Classification:** 60A99 68T99

### References

- [1] Shai Ben-David, Pavel Hrubesss, Shay Moran, Amir Shpilka, and Amir Yehudayoff: Learnability can be undecidable. *Nature Machine Intelligence* **1**, 1 (2019), 44-48.

# Convergence of lattice valued options to their Black-Scholes limit

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## **Abstract:**

Lattice methods, also known as tree methods, are very popular to evaluate the price of security derivatives due to their simplicity and great flexibility. Studying the fine properties of that convergence, allows to construct faster converging trees and/or combine them with other numerical techniques, in order to improve the converge. We present here various ways of achieving this. We show how the error of the method can be expanded in powered of  $1/\sqrt{n}$ , with closed form formula for the coefficients, and how smoothness can be achieved by altering the probability of one single node. We illustrate the effect of these techniques on the speed of convergence.

**Keywords:** Binomial tree, smooth convergence, error expansion, Black-Scholes

# Rings, Monoids and Module Theory

*Special Session Organized by Ayman Badawi*

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14:25-14:50	AA3003	On Valuation Factorization Domains	Andreas Reinhart
14:50-15:15	AA3010	Class(semi)group of Prufer Domains and Atomicity	Richard Erwin Hasenauer
15:15-15:40	AA3029	The Class Group of h-local Prufer Domains	Gyu Whan Chang
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16:25-16:50	AA-3021	Vanishing of Tor over Fiber Products	Sylvia Wiegand
16:50-17:15	AA3082	Idealization of co-multiplication modules	Majid Ali
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9:25-9:50	AA-3051	Simultaneous Interpolation and p-adic Approximation by Integer-valued Polynomials	Sophie Frisch
9:50-10:15	AA-3012	Factorization behavior in Rings of Integer-valued Polynomials over Dedekind Domains.	Roswitha Rissner
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Special Session B2: Rings, Monoids and Module Theory Chair: Sophie Frisch Room:SBA0001			
Time	ID	Title	Speaker
11:00-11:25	AA3058	Tilting Modules and Tilting Torsion Pairs	Alberto Tonolo
11:25-11:50	AA3039	Injective Modules over the Jacobson Algebra	Francesca Mantese
11:50-12:15	AA3056	Minimal Approximation of some Classes of Modules over Commutative Rings	Giovanna Le Gros

<b>Special Session B3: Rings, Monoids and Module Theory</b> <b>Chair: Roger Wiegand</b> <b>Room:SBA0001</b>			
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14:00-14:25	AA-3011	Cohen-Macaulay Unit Graphs of Commutative Rings.	T. Asir
14:25-14:50	AA-3060	Uniqueness of Zero-Divisor Graphs with Loops	Aihua Li
14:50-15:15	AA-3038	BZS Near-Rings and Rings	Mark Farag
<b>Special Session C1: Rings, Monoids and Module Theory</b> <b>Chair: Marco Fontana</b> <b>Room:SBA0001</b>			
Time	ID	Title	Speaker
10:30-10:55	AA-3050	Some Generalizations of Noetherian Rings	Jim Coykendall
10:55-11:20	AA-3018	Locally Free Cancellation for Definite Quaternion Algebras	Daniel Smertnig
11:20-11:45	AA-3035	Algebras whose Group of Units is Hyperbolic	Victor Bovdi
11:45-12:10	AA-3036	A Computation in Reflection Groups	Dong-il Lee
<b>Special Session C2: Rings, Monoids and Module Theory</b> <b>Chair: Jim Coykendall</b> <b>Room:SBA0001</b>			
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14:00-14:25	AA-3042	Factorization of norms in rings of algebraic integers and weighted zero-sum problems	Wolfgang A. Schmid
14:25-14:50	AA-3028	On strongly primary monoids with a focus on puioux monoids	Felix Gotti
14:50-15:15	AA-3030	When is a Puiseux Monoid Atomic?	Marly Gotti
15:15-15:40	AA-3044	On the Notion of Krull Super-dimension	A.N. Zubkov
15:40-16:05	AA-3006	On 2-absorbing Ideals of Commutative Semiring	M. Saleh

## ON $t$ -LOCAL DOMAINS AND VALUATION DOMAINS

Marco Fontana

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*The present talk is based on a joint work with Muhammad Zafrullah (Department of Mathematics, Idaho State University, USA).*

### Abstract:

Recall that a nonzero fractional ideal  $E$  of an integral domain  $D$  is a  $t$ -ideal if

$$E = E^t := \bigcup \{F^v \mid F \subseteq E, F \text{ is a nonzero finitely generated fractional ideal}\}$$

(where  $F^v := (D : (D : F)) = (F^{-1})^{-1}$ ).

In a valuation domain  $(V, M)$  every nonzero finitely generated ideal  $J$  is principal and so, in particular,  $J = J^t = J^v$ , hence the maximal ideal  $M$  is a  $t$ -ideal. Therefore, the  $t$ -local domains, i.e., the local domains, with maximal ideal being a  $t$ -ideal, are “rather close” to valuation domains, but, as we will see in detail, not so close. Indeed, for instance, a localization of a  $t$ -local domain is not necessarily  $t$ -local, but of course a localization of a valuation domain is still a valuation domain.

So it is natural to ask: under what conditions is a  $t$ -local domain a valuation domain? The main purpose of this talk is to address this question, surveying in part previous work by various authors containing useful properties for applying them to our goal.

**Keywords:**  $t$ -operation, local ring, valuation domain.

**2010 Mathematics Subject Classification:** 13G05, 13H10, 13F30

## ON VALUATION FACTORIZATION DOMAINS

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**General area of research:** Algebra and Its Applications

### Abstract:

Let  $D$  be an integral domain and let  $a \in D$  be a nonzero nonunit. Then  $a$  is called a valuation element of  $D$  if  $aV \cap D = aD$  for some valuation overring  $V$  of  $D$ . Furthermore,  $D$  is said to be a valuation factorization domain (VFD) if every nonzero nonunit of  $D$  is a finite product of valuation elements of  $D$ . In this talk we study VFDs and investigate their connections with other well-known concepts. In particular, we show that every VFD is a Schreier domain and prove that every weakly Matlis GCD-domain is a VFD. Moreover, we present characterizations of VFDs and provide several (sufficient) conditions which force a VFD to be a weakly Matlis GCD-domain. Finally, we complement our results by a few counterexamples.

**Keywords:** Valuation element, VFD, weakly Matlis GCD-domain

**2010 Mathematics Subject Classification:** 13A15, 13F05, 13G05

## References

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## Class(semi)group of Prüfer domains and atomicity

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**Abstract:** We explore the connection between atomicity in Prüfer domains and their corresponding class groups. We observe that a class group of infinite order is necessary for almost Dedekind and Prüfer domains of finite character to be atomic. We construct an almost Dedekind domain and explicitly describe its ideal class semigroup.

**Keywords:** Factorization, Prüfer domains

**13A50, 13F15**

## THE CLASS GROUP OF h-LOCAL PRÜFER DOMAINS

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**Abstract:** Let  $G$  be an abelian group. In this talk, we show that there is an h-local Prüfer domain with ideal class group  $G$ . This is a generalization of Claborn's result that every abelian group is the class group of a Dedekind domain.

**Keywords:** h-local Prüfer domain, ideal class group.

**2010 Mathematics Subject Classification:** 13A15, 13F05

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## TORSION IN TENSOR PRODUCTS AND RIGID IDEALS

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**Abstract:** In [1] Huneke and Wiegand made the following conjecture, for a local Gorenstein domain  $R$ : If both  $M$  and the tensor product of  $M$  with its algebraic dual  $M^*$  are maximal Cohen-Macaulay modules, then  $M$  must be free. The conjecture (still open) reduces to the one-dimensional case, where it can be restated as follows: If  $M$  is finitely generated, torsion-free, and rigid, then  $M$  is free. (A module  $M$  is said to be rigid provided every self-extension of  $M$  splits.) In this talk I will describe progress on the conjecture for the case of ideals in one-dimensional domains. This talk is based on [2].

**Keywords:** Gorenstein ring, tensor product, torsion

**2010 Mathematics Subject Classification:** 13D07, 13C14, 13C99

## References

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## VANISHING OF TOR OVER FIBER PRODUCTS

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**Abstract:** Let  $(S, \mathfrak{m}, k)$  and  $(T, \mathfrak{n}, k)$  be local rings, and let  $R$  denote their fiber product over their common residue field  $k$ . Inspired by work of Nasseh and Sather-Wagstaff [2], we explore consequences of vanishing of  $\mathrm{Tor}_m^R(M, N)$  for various positive integers  $m$ , where  $M$  and  $N$  are finitely generated  $R$ -modules. This talk is based on [1].

**Keywords:** fiber product, Tor

**2010 Mathematics Subject Classification:** 13H15

## References

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# IDEALIZATION OF COMULTIPLICATION MODULES

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Sultan Qaboos University

Let  $R$  be a commutative ring with identity and  $M$  an  $R$ -module. The  $R$ -module  $R(M) = R(+)M$  becomes a commutative ring with identity under the product  $(r, m)(s, m') = (rs, sm + rm')$ , called the idealization of  $M$ . The idealization of a module is a well-established method to facilitate interaction between a ring on the one hand and a module over a ring on the other. The basic construction is to embed the module  $M$  as an ideal in a ring  $R(M)$  which contains  $R$  as a subring. This technique was used with great success by Nagata. For a comprehensive survey on idealization, Anderson and Winders, Huckaba can be consulted.  $0(+)M$  is an ideal of  $R(M)$  satisfying  $(0(+)M)^2 = 0$ , and the structure of  $0(+)M$  as an ideal of  $R(M)$  is essentially the same as the  $R$ -module structure of  $M$ . Every ideal contained in  $0(+)M$  has the form  $0(+)N$  for some submodule  $N$  of  $M$ , and every ideal containing  $0(+)M$  has the form  $I(+)M$  for some ideal  $I$  of  $R$ . Prime (maximal) ideals of  $R(M)$  have the form  $P(+)M$  where  $P$  is a prime (maximal) ideal of  $R$ . Homogeneous ideals of  $R(M)$  have the form  $I(+)N$ , where  $I$  is an ideal of  $R$ ,  $N$  a submodule of  $M$  such that  $IM$  is a subset of  $N$ , that is  $[N : M I] = M$ . Ideals of  $R(M)$  need not be homogeneous.

In this talk, we develop the method of idealization particularly in the context of comultiplication modules. We show, for example, that if  $I(+)N$  is a comultiplication ideal of  $R(M)$  then  $I$  is comultiplication. Assuming further that  $M$  is comultiplication then  $N$  is a comultiplication submodule of  $M$ .

Keywords: multiplication module, comultiplication submodule, copure submodule, coideal submodule

AMS (2010) 13C05, 13C13, 13A15.

## References:

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- [6] Anderson, D. D., Winders, M., *Idealization of a module*. Rocky Mountain J. Math. (2009).

## SOME FINITENESS CONDITIONS ON THE SET OF INTERMEDIATE RINGS OF A RING EXTENSION WITH ZERO DIVISORS

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### Abstract:

Many recent works have obtained new and interesting results on the set of intermediate rings in extensions of integral domains satisfying several finiteness conditions. Such results are still in need when we consider rings that are not integral domains. The main aim of this work is to extend such studies for ring extensions  $R \subseteq S$  with nontrivial zero divisors. This study focuses mainly on the cardinality and the length of the sets under investigation. Several equations related to the studied conditions are established. The obtained results generalize several related recent results on the field, however many problems are still open.

**Keywords:** ring extension, intermediate ring, normal pair.

**2010 Mathematics Subject Classification:** 13B02, 13B22, 13E15, 13E99, 13F05, 13G05.

### References

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## A GENTLE INTRODUCTION INTO COGALOIS THEORY

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**Abstract:** Roughly speaking, *Cogalois Theory* investigates field extensions, finite or not, that possess a Cogalois correspondence. This theory is somewhat dual to the very classical *Galois Theory* dealing with field extensions possessing a Galois correspondence. An important part of *Galois Theory* is the classical *Kummer Theory* investigating Galois field extensions, not necessarily finite, having an Abelian Galois group of finite exponent; these extensions possess both a Galois and Cogalois correspondence, and their theory can be completely and easily deduced from *Cogalois Theory*.

The aim of the talk is to present to a general audience some of the basic concepts, results, and applications of this fairly new theory, born about 35 years ago.

**Keywords:** Field Theory, Cogalois Theory, Galois Theory,

**2010 Mathematics Subject Classification:** 12F05, 12F10, 12F99, 12E30

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## SIMULTANEOUS INTERPOLATION AND P-ADIC APPROXIMATION BY INTEGER-VALUED POLYNOMIALS

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**Abstract:** Let  $D$  be a Dedekind domain with finite residue fields and  $\mathcal{F}$  a finite set of maximal ideals of  $D$ . Let  $r_0, \dots, r_n$  be distinct elements of  $D$ , pairwise incongruent modulo  $P^{k_P}$  for each  $P \in \mathcal{F}$ , and  $s_0, \dots, s_n$  arbitrary elements of  $D$ .

We show that there is an interpolating  $P^{k_P}$ -congruence preserving integer-valued polynomial, that is,  $f \in \text{Int}(D) = \{g \in K[x] \mid g(D) \subseteq D\}$  with  $f(r_i) = s_i$  for  $0 \leq i \leq n$ , such that, moreover, the function  $f: D \rightarrow D$  is constant modulo  $P^{k_P}$  on each residue class of  $P^{k_P}$  for all  $P \in \mathcal{F}$ .

**Keywords:** integer-valued polynomials, interpolation, p-adic approximation

**2010 Mathematics Subject Classification:** Primary 13B25; Secondary 12C05, 13F20, 13M10, 11C08, 13F05.

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## FACTORIZATION BEHAVIOR IN RINGS OF INTEGER-VALUED POLYNOMIALS OVER DEDEKIND DOMAINS

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### Abstract:

Non-unique factorization of elements into irreducibles has been observed in the ring of integer-valued polynomials and its generalizations. We show that all finite (multi-)sets of natural numbers greater than 1 occur as (multi-)sets of lengths of a polynomial in  $\text{Int}(D)$  where  $D$  is a Dedekind domain with infinitely many maximal ideals all of whose residue fields are finite. For given integers  $1 < m_1 \leq \dots \leq m_n$  we can explicitly construct an element  $f \in \text{Int}(D)$  which has exactly  $n$  essentially different factorizations of lengths  $m_1, \dots, m_n$ . In this talk, we speak about the construction techniques and consequences.

This is joint work with S. Frisch and S. Nakato.

**Keywords:** factorizations, integer-valued polynomials

**2010 Mathematics Subject Classification:** 13A05, 13B25, 13F20, 11R04, 11C08

## On EM Conditions

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### Abstract

A ring  $R$  is called an EM-ring, if for each  $f(x) = \sum_{i=0}^n a_i x^i \in R[x]$  there exist  $a \in R$  and a regular polynomial  $g = \sum_{i=0}^m b_i x^i$  such that  $f(x) = ag(x)$ . The ring  $R$  is called locally EM-ring if for each prime ideal  $P$  of  $R$ , we have  $R_P$  is an EM-ring. A ring  $R$  is called an EM-Hermite (K-Hermite) ring, if for each  $f(x) = \sum_{i=0}^n a_i x^i \in R[x]$  there exist  $a \in R$  and a polynomial of the same degree  $g = \sum_{i=0}^n b_i x^i$  such that  $f(x) = ag(x)$  and  $c(g)$  is a regular ideal ( $c(g) = R$ ).

In this article we study the implications of these rings and some of their properties. We also relate them to other rings such as PP-rings, Armendariz rings, rings with property A, generalized morphic rings, and a.c condition rings.

**Keywords:** EM-ring, EM-Hermite ring, K-Hermite ring, Bezout ring.

**2010 Mathematics Subject Classification:** 13A, 13B25, 13B30, 13C10

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## Tilting modules and tilting torsion pairs

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**Abstract:** Let  $A$  be a ring and  $A\text{-Mod}$  the category of left  $A$ -modules. The notion of tilting module has been axiomatised in 1979 by Brenner and Butler [1], generalising that of progenerator for modules of projective dimension 1. Such a classical 1-tilting module  $T$  determines a torsion pair  $(KE_0(T), KE_1(T))$  on the category of modules, where

$$KE_i(T) = \{M \in A\text{-Mod} : \text{Ext}^j(T, M) = 0 \ \forall j \neq i\}, \ i = 0, 1.$$

Therefore, each module  $M$  admits a filtration with two factors belonging to  $KE_i(T)$  for  $i = 0, 1$ . Miyashita [3] extended the tilting notion for modules of projective dimension  $n \geq 1$ . A classical  $n$ -tilting module  $T$  naturally gives rise to  $n + 1$  classes of modules in  $A\text{-Mod}$ , called Miyashita classes:

$$KE_i(T) = \{M \in A\text{-Mod} : \text{Ext}^j(T, M) = 0 \ \forall j \neq i\}, \ i = 0, 1, \dots, n.$$

For  $n > 1$ , the Miyashita classes are too small in order to filter every left  $A$ -module. Working on the derived category  $D(A)$ , the Miyashita classes can be equivalently described as

$$KE_i(T) = \{M \in A\text{-Mod} : \text{Hom}_{D(A)}(T, M[j]) = 0 \ \forall j \neq i\}, \ i = 0, 1, \dots, n.$$

Consider the larger classes

$$KE_i(T) = \{M^\bullet \in D(A) : \text{Hom}_{D(A)}(T, M^\bullet[j]) = 0 \ \forall j \neq i\}, \ i = 0, 1, \dots, n.$$

Generalising [2], we provide, for a not necessarily finitely generated  $n$ -tilting module  $T$ , a decomposition of any module in terms of objects in these classes. This decomposition generalises the one found for  $n = 1$ : the Miyashita classes can indeed be regarded as the piece of these new classes visible in the category of modules. For  $i = 1, \dots, n$ , the class  $KE_i(T)$  is the  $(-i)$ -shift of the heart of the  $t$ -structure  $\mathcal{T}$  associated to the tilting module  $T$ . The decomposition of any modules is obtained using  $n$  torsion pairs in the hearts of as many  $t$ -structures linking the natural  $t$ -structure to the  $t$ -structure  $\mathcal{T}$  in  $D(A)$ .

**Keywords:** tilting modules, derived categories, torsion pairs.

**2010 Mathematics Subject Classification:**

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## INJECTIVE MODULES OVER THE JACOBSON ALGEBRA

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**Abstract:** The Jacobson algebra  $R$  is the algebra  $K\langle X, Y \mid XY = 1 \rangle$ . It possesses interesting representation-theoretic and ring-theoretic properties, and close connections between several long standing conjectures in mathematics as, for instance, the Direct Finiteness Conjecture of Kaplansky. In particular, in [1] the authors suggested an approach to the Direct Finiteness Conjecture based on the module theory of  $R$ .

The Jacobson algebra  $R$  is isomorphic to the Leavitt path algebra  $L_K(\mathcal{T})$ , where  $\mathcal{T}$  is known as the Toeplitz graph. In [2] and [3] the authors developed tools and techniques to study simple modules and their injective envelopes in any Leavitt path algebras. Applying these ideas we are able to give a complete classification of the injective envelopes of the simple modules and an explicit description of an injective cogenerator in  $R$ , leading to a better understanding of the representation theory of the Jacobson algebra.

**Keywords:** Leavitt path algebras, simple modules, Prüfer modules

**2010 Mathematics Subject Classification:** 16D10, 16D50, 16S10

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## MINIMAL APPROXIMATIONS OF SOME CLASSES OF MODULES OVER COMMUTATIVE RINGS

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**Abstract:** We consider cotorsion pairs  $(\mathcal{A}, \mathcal{B})$  generated by classes of modules of projective dimension at most one over commutative rings  $R$ . We are interested in when these cotorsion pairs admit covers or envelopes. We investigate Enochs's Conjecture in this setting, that is the question of whether  $\mathcal{A}$  is covering necessarily implies that  $\mathcal{A}$  is closed under direct limits.

We show that the class  $\mathcal{P}_1(R)$  of modules of projective dimension at most one over a semihereditary ring  $R$ , is covering if and only if it is closed under direct limits, giving an example of a cotorsion pair not of finite type which satisfies Enochsconjecture.

For the case of cotorsion pairs of finite type, specifically 1-tilting cotorsion pairs  $(\mathcal{A}, \mathcal{T})$  we rely on work of Hrbek [1] and Bazzoni-Positselski [2]. We show that if  $\mathcal{T}$  is enveloping then the associated Gabriel topology  $\mathcal{G}$  must arise from a perfect localisation, that is a flat ring epimorphism  $R \rightarrow R_{\mathcal{G}}$ . Furthermore,  $\mathcal{T}$  is enveloping if and only if the projective dimension (p.dim) of  $R_{\mathcal{G}}$  is at most one and  $R/J$  is a perfect ring for every ideal  $J \in \mathcal{G}$ , if and only if  $\text{p.dim } R_{\mathcal{G}} \leq 1$  and the topological ring  $\text{End}(R_{\mathcal{G}}/R)$  is pro-perfect.

Next, we prove that  $\mathcal{A}$  is a covering class if and only if  $\text{p.dim } R_{\mathcal{G}} \leq 1$  and the localisation  $R_{\mathcal{G}}$  is a perfect ring as well as  $R/J$  for every  $J \in \mathcal{G}$ , hence  $\mathcal{A}$  is closed under direct limits.

**Keywords:** covers, envelopes

**13B30, 13C60, 13D07, 18E40:**

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## COHEN–MACAULAY UNIT GRAPHS OF COMMUTATIVE RINGS

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**Abstract:** Let  $R$  be a finite commutative ring with nonzero identity. The unit graph of  $R$  is the graph in which the vertex set is  $R$ , and two distinct vertices  $x$  and  $y$  are adjacent if and only if  $x+y$  is a unit in  $R$ . In this talk, we determine when the unit graphs are well-covered, and then, by applying the well-coveredness result, we characterize the unit graphs whose edge rings are Cohen–Macaulay.

**Keywords:** Unit graph of a ring, Well-covered graph, Cohen–Macaulay graph.

**2010 Mathematics Subject Classification:** Primary: 05C75, 13H10; Secondary: 05E40

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## Uniqueness of Zero-Divisor Graphs with Loops

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**Abstract:** It is known that rings which have isomorphic zero-divisor graphs are not necessarily isomorphic. Zero-divisor graphs for rings were originally defined without loops because edges are only defined on pairs of distinct nonzero zero-divisors. In this paper, we study zero-divisor graphs of a ring  $R$  that may have loops. We denote such graphs by  $\Gamma_0(R)$ . If  $R$  is a noncommutative ring,  $\vec{\Gamma}_0(R)$  denotes the directed zero-divisor graph of  $R$  that allow loops. Consider two sets of finite rings:  $\{R_1, R_2, \dots, R_m\}$  and  $\{S_1, S_2, \dots, S_t\}$ , where each of the  $R_i$  or  $S_j$  is either a finite field or of the form of  $\mathbf{Z}_{p^\alpha}$  with  $p$  being a prime number and  $\alpha$  being a positive integer. Suppose that  $R \cong R_1 \times R_2 \times \dots \times R_m$ ,  $S \cong S_1 \times S_2 \times \dots \times S_t$ , and neither  $R$  nor  $S$  is a finite field. We show that if  $\Gamma_0(R) \cong \Gamma_0(S)$ , then  $R \cong S$ . We further investigate directed zero-divisor graphs with loops of upper triangular matrices over finite fields. We claim that if  $R$  and  $S$  are two  $n$  by  $n$  upper triangular matrices over finite fields such that  $\vec{\Gamma}_0(R) \cong \vec{\Gamma}_0(S)$ , then  $R \cong S$ .

**Keywords:** Zero-divisor graph with loops, finite ring, direct product.

**2010 Mathematics Subject Classification:** Primary 13A99, 16B99, 20M99; Secondary 05C99.

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## BZS NEAR-RINGS AND RINGS

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### Abstract:

A near-ring  $(N, +, \cdot)$  is said to be BZS, or Boolean - zero square, if given  $n \in N$ , either  $n^2 = n$  or  $n^2 = 0$ . BZS near-rings generalize several previously studied classes of near-rings, including Boolean, zero square, and Malone trivial near-rings.

In the first part of this talk, we discuss the general structure of BZS near-rings as studied in [1]. In particular, we show that the general BZS near-ring case admits properly BZS (i.e., neither Boolean nor zero square) near-rings with additive groups other than elementary abelian 2-groups. When the additive group of a non-Boolean BZS near-ring is of prime order, we find that the near-ring must be zero-symmetric and, in fact, a Malone trivial near-ring. This leads to results giving: i) the number of multiplications on the additive group  $(\mathbb{Z}_p, +)$ , for  $p$  a prime yielding a non-Boolean BZS near-ring, and ii) the number of isomorphism classes of such near-rings. We also discuss results showing that, although the set of all nilpotent elements of a BZS near-ring  $N$ ,  $\text{nil}(N)$ , need not be an ideal of  $N$ ,  $\text{nil}(N)$ , is: a) a subsemigroup of  $N$  with respect to multiplication, and b) an ideal precisely when it is a left ideal of  $N$ . Finally, we show that non-Boolean BZS near-rings of prime order have trivial centers provided their multiplication is not identically zero.

In the second part of the talk, we investigate the structure of a BZS ring  $(R, +, \cdot)$  as studied in [2]. After giving nontrivial examples of BZS rings, we discuss results showing that: a) when  $R$  is properly BZS, its additive group is isomorphic to a direct product of copies of  $\mathbb{Z}_2$ , b) the set of all nilpotent elements  $\text{nil}(R)$  is always an ideal of  $R$ , c)  $R$  properly BZS implies  $\text{nil}(R)$  is the unique maximal ideal of  $R$ ,  $\text{nil}(R)$  has index 2 as an ideal of  $R$ , and it is the only prime ideal of  $R$ , and d) we find, up to isomorphism, all properly BZS rings on the additive group  $\mathbb{Z}_2 \oplus \mathbb{Z}_2$ .

**Keywords:** Boolean, zero square, BZS, ring, near-ring.

**2010 Mathematics Subject Classification:** 16U99, 16Y30

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## SOME GENERALIZATIONS OF NOETHERIAN RINGS

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**Abstract:** The Noetherian condition is one of the most central and useful concepts in commutative algebra. At least part of its utility stems from the fact that if  $R$  is a Noetherian ring, then so is its polynomial extension  $(R[x])$ , its power series extension  $(R[[x]])$ , and localizations  $(R_S)$ . There are many generalization of the Noetherian property (e.g coherence, Noetherian prime spectrum, and the SFT property), and in this talk, we will explore some of these generalizations (in particular the strong and very strong finite-type conditions that are important in the Krull dimension theory of formal power series rings). The focus in this talk will be on the stability of these properties in extension rings.

**Keywords:** Noetherian rings, SFT property, power series.

**2010 Mathematics Subject Classification:** 13E05, 13F20

## LOCALLY FREE CANCELLATION FOR DEFINITE QUATERNION ALGEBRAS

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**Abstract:** Let  $B$  be a central simple algebra over a number field, and let  $O$  be an order in  $B$ . Stable isomorphism classes of finitely generated, locally free  $O$ -modules can be described in terms of their rank and a (one-sided) ideal class, similar to Steinitz's theorem. If strong approximation holds, one has locally free cancellation, meaning that  $M \oplus K \cong N \oplus K$  implies  $M \cong N$ . This implies that the characterization by rank and class is in fact up to isomorphism. However, for definite quaternion algebras, in general, locally free cancellation fails. I will talk about the classification of the finitely many definite quaternion orders that still possess locally free cancellation.

**2010 Mathematics Subject Classification:** Primary 11R52; Secondary 11E41, 11Y40, 16G30, 16H20

## ALGEBRAS WHOSE GROUP OF UNITS IS HYPERBOLIC

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### Abstract:

In the articles [3, 4, 5] the structure of a group  $G$  for which the group of units  $U(KG)$  of the group ring  $KG$  over a commutative ring  $K$  is a hyperbolic group (in the sense of M. Gromov [2]) was considered. A complete description of the structure was given only in [1]. We try to extend the results and technique of [1] to certain classes of algebras whose group of units is hyperbolic.

**Keywords:** group algebra, crossed product, hyperbolic group

**2010 Mathematics Subject Classification:** 16S34, 16U60, 20F67

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## A COMPUTATION IN REFLECTION GROUPS

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**Abstract:** A couple of decades have passed since we first obtained a computational result on the structure for reflection groups and their deformation algebras [1, 4]. Several effective tools were developed and applied to various contexts. In this presentation, we introduce a computational background and some computations in the setting of complex reflection groups [2, 3, 5, 6]. If time permits, some issues for Shephard groups of a particular type will be discussed.

**Keywords:** complex reflection group, Shephard group.

**2010 Mathematics Subject Classification:** 20F55, 05E15.

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<sup>1</sup>This research was supported by NRF Grant # 2018R1D1A1B07044111.

## FACTORIZATIONS OF NORMS IN RINGS OF ALGEBRAIC INTEGERS AND WEIGHTED ZERO-SUM PROBLEMS

Wolfgang A. Schmid

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**General area of research:** Algebra and Its Applications; Session: Rings, Monoids and Module Theory

**Abstract:** Let  $K$  be an algebraic number field and let  $R_K$  denote its ring of algebraic integers. For  $a \in R_K$  let  $N(a)$  denote the absolute norm of  $a$ , that is the ideal norm of the principal ideal generated by  $a$ . The set  $H_N$  of all integers that are the norm of some non-zero element of  $R_K$ . The set  $H_N$  is a multiplicative submonoid of the positive integers.

It is easy to see that this monoid is an atomic monoid, that is, every element is the product of irreducible elements. The purpose of this talk is to study the arithmetic of this type of monoids.

We show that for quadratic number fields the arithmetic of this monoid is essentially the same, technically there is a transfer-homomorphism, as the arithmetic of the monoid of plus-minus weighted zero-sum sequences over the ideal class group. We recall that a different link between norms of algebraic integers and weighted zero-sum problems was recently explored by Halter-Koch [1].

We use this connection to establish some explicit results on the arithmetic of these monoids. Generalizations for number fields of higher degree are discussed.

This is joint work with S. Boukheche, K. Merito and O. Ordaz.

**Keywords:** Factorization theory, algebraic integer, norm, zero-sum sequence

**2010 Mathematics Subject Classification:** 13F15, 11R27, 11B30

## References

- [1] F. Halter-Koch. Arithmetical interpretation of weighted Davenport constants. *Archiv der Mathematik* 103 (2014), 125–131.

## ON STRONGLY PRIMARY MONOIDS, WITH A FOCUS ON PUISEUX MONOIDS

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**Abstract:** Primary and strongly primary monoids play an important role in the ideal and factorization theory of commutative monoids and integral domains [3, 4]. Additive submonoids of the nonnegative rationals, also known as Puiseux monoids, are primary monoids. Puiseux monoids have earned significant attention recently because of their connections with and applications to numerical monoids [1, 5], factorization theory [7, 8], and commutative algebra [2, 6]. Here we discuss necessary and sufficient conditions for a Puiseux monoid to be strongly primary. Then we present arithmetic and algebraic characterizations of when a Puiseux monoid is globally tame.

**Keywords:** strongly primary monoids, Puiseux monoids, tameness

## References

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- [6] F. Gotti: *Increasing positive monoids of ordered fields are FF-monoids*, J. Algebra **518** (2019), 40–56.
- [7] F. Gotti: *Puiseux monoids and transfer homomorphisms*, J. Algebra **516** (2018), 95–114.
- [8] F. Gotti: *Systems of sets of lengths of Puiseux monoids*, J. Pure Appl. Algebra **223** (2019), 1856–1868.

## WHEN IS A PUISEUX MONOID ATOMIC?

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**Abstract:** A Puiseux monoid is an additive submonoid of the nonnegative cone of  $\mathbb{Q}$ . If  $M$  is a Puiseux monoid, then the question of when each nonunit element of  $M$  can be written as a sum of irreducible elements (or is *atomic*) is surprisingly difficult. For instance, although various techniques have been developed over the past few years to identify subclasses of Puiseux monoids which are atomic, no general characterization of such monoids is known. Here we discuss some of the most relevant aspects related to the atomicity of Puiseux monoids. We provide characterizations of when  $M$  is finitely generated, factorial, half-factorial, other-half-factorial, Prüfer, seminormal, root-closed, and completely integrally closed. In addition to the atomic property, precise characterizations are also not known for when  $M$  satisfies the ACCP, is a BF-monoid, or is an FF-monoid; in each of these four cases, we construct classes of Puiseux monoids satisfying these properties.

**Keywords:** Puiseux monoids, atomicity, factorization theory, Prüfer monoids, numerical monoids, ACCP, BF-monoids, FF-monoids.

**2010 Mathematics Subject Classification:** Primary: 20M13; Secondary: 06F05, 20M14

## References

- [1] S. T. Chapman, F. Gotti, and M. Gotti, *Factorization invariants of Puiseux monoids generated by geometric sequences*, Comm. Algebra (2019), <https://doi.org/10.1080/00927872.2019.1646269>
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## ON THE NOTION OF KRULL SUPER-DIMENSION

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**General area of research:** Algebra

**Abstract:** The notion of Krull super-dimension of a super-commutative super-ring is introduced. This notion is used to describe regular super-rings and calculate Krull super-dimensions of completions of super-rings. Moreover, we use this notion to introduce the notion of super-dimension of any irreducible superscheme of finite type. Finally, we describe nonsingular superschemes in terms of sheaves of Kähler superdifferentials.

This is the joint work with A.Masuoka (Tsukuba University, Japan).

**Keywords:** super-commutative super-ring, superscheme, Krull super-dimension

**2010 Mathematics Subject Classification:** 20G40, 20G05

## References

- [1] A.Masuoka and A. N. Zubkov, On the notion of Krull super-dimension, to appear in Journal of Pure and Applied Algebra.

## ON 2- ABSORBING IDEALS OF COMMUTATIVE SEMIRING

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and *msaleh@birzeit.edu*

### Abstract:

Let  $S$  be a commutative semiring with unity different than zero. In this research, we study the concept of 2-absorbing ideal of  $S$  which can be considered as a genralization of prime ideals. It is shown that the radical of 2-absorbing ideal is also 2-absorbing ideal and there are at most 2 prime  $k$ - ideals of  $S$  that are minimal over a 2- absorbing ideals. We introduce some of its basic characteristics which are analogue to commutative ring.

**Keywords:** Semiring, prime ideal, 2-absorbing ideal.

### References

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# Statistical Learning-Data Mining-Probability

*Special Session Organized by Hana Sulieman*

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Special Session A2: Statistical Learning-Data Mining-Probability Chair: Mahmoud Awad Room:SBA0009			
Time	ID	Title	Speaker
16:00-16:25	ST-3018	Travelers' Perception of Service Quality at Dubai International Airport	Mahmoud Awad
16:25-16:50	ST-3023	An Artificial Neural Network Approach to Nonparametric Control Chart	Nimbale S. M
16:50-17:15	ST-3004	A Hybrid Artificial Neural Networks and Arima Models for Forecasting Electricity Consumption in Palestine	Samir K. Safi
17:15-17:40	MD-3003	Design New Pseudorandom Number Generators using Tabu Programming	Emad Mabrouk
17:40-18:05	ST-3017	Human Gut Microbiota Composition and Functionality Correlation with Age	Mohammad Tahseen Al Bataineh

Special Session C1: Statistical Learning-Data Mining-Probability Chair: Linda Ismail Room:SBA0008			
Time	ID	Title	Speaker
10:30-10:55	ST-3033	Evaluating Extreme Natural Hazards: An Application to Cyclones	G. De Masi
10:55-11:20	ST-3028	Fuzzy Time Series Forecasting Model Based on Singular Spectrum Analysis Decomposition	Subanar
11:20-11:45	AM-3024	Inference In Bayesian Networks: Junction Trees Constructions	Linda Smail

## TRAVELERS' PERCEPTION OF SERVICE QUALITY AT DUBAI INTERNATIONAL AIRPORT

**Mahmoud Awad, Ayman Alzaatreh, Alia AlMutawa, Hind Al Ghumlasi and Mariam Almarzooqi**

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### **Abstract:**

Dubai's aviation industry is one of the most leading global aviation centers with a customer centric focus and tremendous challenges. The purpose of this paper is to evaluate the perception of travelers of Dubai International Airport (DXB) terminal 3 relative to the services provided by the airport and propose actions to improve the customer journey. Travelers feedback was captured through interviews, initial survey, online review, and literature review and a more focused survey is developed as the main quantitative research tool. A confirmative factor analysis supported by structural equation modeling was used and revealed check in, assurance and empathy, and availability to some extent as the main areas impacting travelers satisfaction and impression of DXB. Moreover, Kruskal-Wallis test suggested that nationality impact travelers experience at the airport. Finally, arrival-departure curves are used to provide suggestions for check-in process improvement.

**Keywords:** Airport Experience; Service Quality; Passenger segmentation

**2010 Mathematics Subject Classification:** 62

## AN ARTIFICIAL NEURAL NETWORK APPROACH TO NONPARAMETRIC CONTROL CHART

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### **Abstract:**

To achieve high quality demand by customers and be competitive in global market, major interest of industrial researcher to interface statistical process control (SPC) with computerized manufacturing techniques. In recent years many of researcher demonstrated artificial neural network (ANN) models as automate task of interpreting the control chart. In this paper, ANN model is developed for monitoring the shift in process location when underlying distribution is not known exactly. Distribution-free or nonparametric control charts can be useful when the underlying process distribution is not much known or can be assumed. The sign chart based on the sign statistic is well known and most frequently used nonparametric control chart for monitoring location of process. The average run length (ARL) performance of the proposed ANN model is evaluated through a simulation study and compared with the corresponding nonparametric sign control chart. The study indicates that the proposed ANN model is more efficient than traditional nonparametric sign control chart for detecting the shifts in process location.

**Keywords:** *Artificial neural network, sign control chart, average run length.*

### **2010 Mathematics Subject Classification: Statistics**

### **References**

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## A hybrid Artificial Neural Networks and ARIMA Models for Forecasting Electricity Consumption in Palestine

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### **Abstract:**

The Problem of electricity consumption in Gaza Strip increases as the gap between supply and demand increases over the last ten years, and resulting on significant impact on different aspects of the economic status in Gaza Strip. This paper aims to build a robust and reliable forecasting model with a Hybrid Artificial Neural Networks (ANNs) and Autoregressive Integrated Moving Average (ARIMA). Researchers tend to add more and more variables in the proposed forecasting model. Does a more complex model necessarily do a better forecasting than a simpler one? We compare the forecasting performance for these models on real monthly data for electricity consumption in Palestine in the period 2000-2019. Different forecasting approaches and criteria to choosing a forecasting technique will be discussed in this paper. The selected best forecasting models will be compared using different forecasting criteria.

**Keywords:** Hybrid model, Neural Networks, ARIMA, Time Series Forecasting.

**2010 Mathematics Subject Classification:** Statistics

### **References**

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## DESIGN NEW PSEUDORANDOM NUMBER GENERATORS USING TABU PROGRAMMING

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### **Abstract:**

Random number generators play a key role in numerous algorithms of computer science and security. There are two main types of devices used to generate random numbers, physical devices called True Random Number Generators (TRNGs) and computational devices called Pseudorandom Number Generators (PRNGs) [3,4,5]. A TRNG is mainly based on measuring some physical phenomena that seem to have random behaviors. On the other hand, a PRNG uses a computational algorithm to produce long sequences of results that are regarded as random. Because of the high cost of hardware implementations of TRNGs, it is important to develop powerful and efficient PRNGs that can be implemented in hardware and software.

In this paper, the Tabu Programming (TP) algorithm is used to discover a set of highly nonlinear functions that can be used as cores for efficient cryptoquality PRNGs. The TP algorithm incorporates the search strategy of the Tabu Search method with the tree data structure of the Genetic Programming (GP) algorithm to deal with computer programs as solutions for a given problem [1,2]. Through a set of numerical experiments, the TP algorithm is shown to generate a set of highly nonlinear functions with desired security properties. Moreover, computational results show that the TP algorithm compares favorably to the GP implementations for the same problem.

**Keywords:** Pseudorandom Number Generator, Security, Tabu Programming.

**2010 Mathematics Subject Classification:** 68T05, 68T20

### **References:**

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## HUMAN GUT MICROBIOTA COMPOSITION AND FUNCTIONALITY CORRELATION WITH AGE

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### Abstract:

Ageing is a natural process with constant changes in the structure, composition and function of body organs and systems. Among the several compositional changes, human gut microbiome, a complex community of trillions of microorganisms also demonstrate noticeable shifts with age. Mounting evidence indicates that these alterations of gut microbiota composition may influence age related health and diseases in the host. However, the precise changes in gut microbiota composition and its role in ageing process remains unknown. In this study, we performed 16 S rRNA- and ITS2 based microbial profiling analysis of 51 stool samples from young and old individuals. Remarkably, the gut microbiota of elderly individuals was shown to be depleted of members belonging to *Prevotella*, *Megamonas*, *Firmicutes*, and *Sutterella*. In contrast, elderly individuals have significant fungal over-representation. Moreover, the metabolic pathways of the gut microbiomes of ten individuals were evaluated through shotgun metagenomics using MiSeq platform, demonstrating that elderly are depleted in metabolic pathways involved in short-chain fatty acids and saccharolytic potential as compared to young individuals. The identified differences in bacterial and fungal composition and metabolic capabilities provides important information that may eventually lead to the development of novel and more effective management strategies to treat age-related illnesses.

**Keywords:** Ageing, Microbiome, Gut, statistics Education, Metabolic capabilities

## EVALUATING EXTREME NATURAL HAZARDS: AN APPLICATION TO CYCLONES

**G. De Masi**

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### **Abstract:**

In regions exposed to cyclonic phenomena, structures near the shore line are threaded by the impact of cyclones. For this reason is critical to assess the extreme cyclone expected during the lifetime of the structure. Following best practices, it is common to provide the extreme expected cyclones associated to some relevant return periods.

Cyclonic phenomena are very complex, being characterized by different tracks, different wind velocity during cyclone evolution, different wind field distributions, different durations. During the design of structure at a particular location, determining the cyclone associated to a certain return period at that site remains an hard task [1]. Nevertheless, it is a crucial issue of fundamental importance from the safety point of view, given the economical and environmental impact of possible damages, considering also an apparent increasing strength of cyclones, probably due to climate change[2].

Estimation is difficult, however, for at least two reasons. The first is related to recorded data mainly referring to previous decades. The estimate of extremes over very long periods is heavily influenced by the small number of severe events that have occurred in recent history. The second concerns the spatial variability of recorded cyclone tracks. Because of the low frequency of cyclone occurrence, estimates of extremes made from a limited database can vary substantially over relatively small distances, even within a spatially limited region where it would be reasonable to expect homogeneous values. Statistical uncertainty grows exponentially, moving toward cyclonic peripheral areas affected very unfrequently by cyclone passages. The commonly used methods to account for this uncertainty are historical track shifting and deductive approaches, but a certain degree of subjectivity is intrinsic in these approaches.

For this reason, here a more proper statistical methodology is proposed, to account for cyclone spatial variability on extreme evaluations. It is based on the Inverse First Order Reliability Method (IFORM) used to determine the bivariate exceedance probability of the wind speed and distance from the coast for cyclonic events in any selected location.

**Keywords:** Statistics, Extremes, Environmental hazards.

**2010 Mathematics Subject Classification:** 60G70 (Extreme value theory; extremal stochastic processes), 62H10 (Multivariate distribution of statistics), 86A05 (Hydrology, hydrography, oceanography)

### **References**

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## FUZZY TIME SERIES FORECASTING MODEL BASED ON SINGULAR SPECTRUM ANALYSIS DECOMPOSITION

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### Abstract:

This study aims to present the decomposition based method for enhancing forecasting accuracy. Inspired by [1], a hybrid fuzzy time series model is established based on several aggregate components obtained by singular spectrum analysis (SSA) decomposition. We implemented four types of first order fuzzy time series, Chen's [2], Yu's [3], Cheng's [4], and Lee's [5] to model the components. We work with an hourly electricity load data series to show that the proposed method can handle the complex pattern in the data. The result shows that the proposed method outperforms SSA with linear recurrent formula (LRF) and fuzzy time series model.

**Keywords:** Fuzzy, SSA, hybrid

**2010 Mathematics Subject Classification:** 62M10, 37M10

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## INFERENCE IN BAYESIAN NETWORKS: JUNCTION TREES CONSTRUCTIONS

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### Abstract:

In Bayesian networks, the computation of probability or conditional probability distributions may require summations relative to very large subsets of variables, in other words summation with respect to a large tree-width of the graph. Consequently there is need to segment, if possible, these computations into several computations that are less intensive and more accessible to parallel treatment. These segmentations are related to the graphic properties of the Bayesian networks.

The correspondence between the graphical structure and the associated probabilistic structure bring many of the problems with inference to graphs problems. However these problems are relatively complex and give rise to a lot of research [1, 2, 3, 5].

Lauritzen and Spiegelhalter presented [4] an algorithm called the LS algorithm that makes use of a clique-tree. The basic idea is to first transform the Bayesian network into a clique-tree, called a junction tree, and then in its second phase, run a message-passing procedure that will transfer, by the end, the potential stored in each clique to the marginal probability of the variables in that clique.

The junction tree shows the important relationship between graph theory and efficient probabilistic inference through a very important and interesting mathematical property of junction trees, the running intersection property.

This work examines an alternative method for constructing junction trees that is essential for the efficient computations of probabilistic enquiries posed on Bayesian networks. It presents a new method for converting a sequence of subsets in a Bayesian network into a proper set of cliques satisfying the running intersection property. We propose an algorithm that allows a sequence of cliques possessing the running intersection property to be built. The obtained set of cliques and separators coincide with the junction tree obtained by the moralization and triangulation process, but it has the advantage of adapting to any computational task by adding links to the graph.

**Keywords:** Bayesian Networks, Inference, Junction Trees.

**2010 Mathematics Subject Classification:** 62F15

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# Statistics

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<b>Session A1: Statistics</b> <b>Chair: Sana Louhichi</b> <b>Room:SBA0009</b>			
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14:00-14:25	ST-3003	Nonparametric Estimation for the Hazard Function	Mounir Arfi
14:25-14:50	ST-3007	On Smoothing Parameters Selection Problems in Nonparametric Regression Models with Dependent Errors	Sana Louhichi
14:50-15:15	ST-3015	Smooth Nonparametric Regression under Shape Restrictions	Hongbin Guo
15:15-15:40	ST-3016	Value-At-Risk Prediction through Vine Copula	Arief Hakim

<b>Session C1: Statistics</b> <b>Chair: Fadlalla Elfadaly</b> <b>Room:SBA0009</b>			
Time	ID	Title	Speaker
10:30-10:55	ST-3011	Frequency Polygon Estimator of the Mode of a Density Function under Weak Dependence	Ahmad Younso
10:55-11:20	ST-3024	On Efficiency of Split-Plot Response Surface Designs when some Observations are Missing	Yisa Yakubu
11:20-11:45	ST-3026	Locally Correct Confidence Intervals for a Binomial Proportion	Fadlalla Elfadaly
11:45-12:10	ST-3005	Skewed-Kotz Distribution with Application to Financial Stock Returns	Amadou Sarr

<b>Session C2: Statistics</b> <b>Chair: Rafiq Hijazi</b> <b>Room:SBA0009</b>			
Time	ID	Title	Speaker
14:00-14:25	ST-3013	Assessment Practices in the Undergraduate Statistics Programs in the Arab World	Rafiq Hijazi
14:25-14:50	ST-3027	Modification of Generalized Space-Time Autoregressive Model for Prediction Monthly Incidence Rate in Banyumas Regency, Indonesia	Nunung Nurhayati
14:50-15:15	ST-3030	Multiple Regression Model to Examine the Incidence of Government Expenditure on Neonatal Mortality in Nigeria	Abubakar Usman
15:15-15:40	ST-3022	Zero-Inflated Models Application to Maternal Mortality Data	Kassim Tawiah

## NONPARAMETRIC ESTIMATION FOR THE HAZARD FUNCTION

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**General area of research: Nonparametric Statistics**

**Abstract:** In mainstream biostatistics, ages at onset are generally referred to as failure time. Failure times are usually observed in clinical trials or cohort studies and often censored. The hazard function has been estimated showing that smaller errors and much less sample variability can be reached. The almost sure convergence of the kernel type estimator of the hazard function is obtained under  $\tilde{\rho}$ -mixing condition with censored data over a sequence of compact sets which increases to  $\mathbb{R}^d$ .

**Keywords: Censored data; Hazard function; Kernel estimation**

**2010 Mathematics Subject Classification: 62G05; 62G07; 62G08**

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## ON SMOOTHING PARAMETERS SELECTION PROBLEMS IN NONPARAMETRIC REGRESSION MODELS WITH DEPENDENT ERRORS

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### Abstract:

This talk is around nonparametric regression, (known also as "learning a function" in machine learning). The purpose of nonparametric regression is to describe and to analyse the trend between a response variable and one or more predictors. This subject was studied by several authors since 1964 and is still relevant, this is due to the fact that nonparametric regression has a lot of applications in different fields (such as in economics, medicine, biology, physics, environment, social sciences and the list is non-exhaustive, see for instance the book [4]).

In this talk, we are interested by kernel nonparametric estimations. These estimations depend on some smoothing parameter  $h$ . Some ways of choosing  $h$  are then needed. The first criterion is known as the Cross Validation criterion which was extended to a Generalized Cross-Validation (GCV) criterion. The GCV has different forms. We refer the reader to [3] who studied this problem in the case of independent observations. Independence of the observations is, however, not a realistic modeling of observed data. In fact, in practice, the data are often correlated. Autoregressive models, autoregressive conditional heteroscedasticity models, Markov chains are example of dependent models (see for instance [2]).

We focus, in this talk, on the case of kernel nonparametric models with dependent errors, more precisely, the case when the errors form a stationary sequence of martingale difference random variables. This case is promising for the study of general stationary dependent errors. We compare the behaviors of the smoothing bandwidths obtained by minimizing three criteria: the average square error, the mean average square error and a Mallows-type criterion adapted to our dependent case. We prove that these three minimizers are nearly equivalent. We give also a normal asymptotic behavior of the gap between the minimizer of the average square error and that of the Mallows-type criterion. We, finally, apply our results to a specific case of martingale difference sequences which is the Autoregressive Conditional Heteroscedasticity (ARCH(1)) processes. The adaptation to the dependent case from the independent one is not trivial and it needs to establish more theoretical and technical results such as maximal inequalities or limit theorems for quadratic forms of dependent data.

This talk is based on our joint paper [1] which was developed in the framework of Grenoble Alpes Data Institute (ANR-15-IDEX-02).

**Keywords:** Nonparametric. Regression. Selection.

**2010 Mathematics Subject Classification:** 62G08. 62G20. 60G10.

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## SMOOTH NONPARAMETRIC REGRESSION UNDER SHAPE RESTRICTIONS

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### Abstract :

Estimation of a function under shape restriction is of considerable interest in many practical applications. It is not uncommon that in many fields, researchers are in the position of having strong presumptions about certain relationships satisfying qualitative restrictions, such as monotonicity and convexity (concavity). Typical examples include the study of utility functions, cost functions, and profit functions in economics (Gallant, 1984; Terrell, 1996), the study of dose response curve in medicine, growth curves of animals and plants in ecology and the estimation of the hazard rate in survival analysis (Chang et al., 2007). Imposing shape-restrictions can improve the predictive performance and reduce overfitting, if the underlying regression function takes the specific form. The classic least squared solutions for shape-restricted estimation are typically neither smooth nor parsimonious. There has been many researches pursuing smooth shape-restricted regressors in recent years (Wang&Ghosh 2012, Mayer, 2008, etc.).

We propose a new non-parametric estimator for univariate regression subject to monotonicity, convexity and concavity constraints with simple structures, by replacing the discrete measures in the non-smooth least squared solutions with continuous ones. Our estimator is composed as the linear combinations of several constructed component functions which satisfy corresponding shape constraints. The smoothness of our model is controlled by one tuning parameter. A fast gradient-based iterative algorithm is used to find the least square estimate with efficiency (Wang, 2007). Asymptotic properties including the consistency of both the estimator and its derivatives have been investigated. Numerical studies show that our estimator is having a better predictive performance comparing to other shape-restricted estimators in most scenarios.

**Key Words:** convex regression; monotone regression; nonparametric; gradient-based algorithm; consistency of derivatives

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## VALUE-AT-RISK PREDICTION THROUGH VINE COPULA

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### Abstract:

Value-at-Risk (VaR) prediction may be applied to either a single random loss or a dependent random loss, given (an)other random loss(es). In this paper, we consider a dependent loss model and carry out a VaR prediction. Our dependent structure is constructed via vine Copula. As for application, we use an aggregate model commonly used in finance and actuarial literatures.

**Keywords:** Aggregate model, dependence, risk measure.

**2010 Mathematics Subject Classification:** 60G25, 62M20, 91B30

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\* Presenter

\*\* Corresponding Author

## FREQUENCY POLYGON ESTIMATOR OF THE MODE OF A DENSITY FUNCTION UNDER WEAK DEPENDENCE

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☒ Oral presentation      ☐ Poster Presentation

**Abstract:** The knowledge of modes of a density function is of great interest in many areas. For example, in unsupervised problems where modes are used as measure of typicality of a set of data. In particular, in modern applications, mode estimation is often used in clustering, with the modes representing cluster centers. There is an extensive literature on mode estimation in the independent case, see the key references: [5], [3], [7], [2], [6] and the references therein. The common approaches consist of estimating the density mode by maximizing an estimate of the unknown density (usually a kernel estimate) on  $\mathbb{R}^d$  or  $\mathbb{R}$ . [?] deal with a simple estimate of the mode by maximizing the kernel density estimate on data. Most of the existing works are concerned with the consistency of the estimators and rates achievable by various approaches. Despite the easy computation, there is only a very few literature dedicated to frequency polygons estimator of density function. This estimator is constructed by connecting with straight lines the mid-bin values of a histogram, for a comprehensive overview, see [8] in the independent case and [4] and [9] in the dependent case. [8] shows that the frequency polygon has rates of convergence similar to those of non-negative kernel estimators with respect to the criterion of integrated mean squared error. [4] extends the results of [8] to the weakly dependent case and [9] investigate the uniform strong consistency of frequency polygon under strong mixing samples. It is important to note that all the asymptotic results on the frequency polygon estimator of density are obtained on the real line. In this paper, we consider the problem of estimating the mode of an unimodal density by maximizing the frequency polygon estimate of the density on data. we consider the simple estimator of the mode of a density function using the frequency polygon estimate. We investigate strong consistency of the estimator for strong mixing sequence of real variables under mild assumptions. Although the results are obtained on the real line but without any differentiability condition on the density.

**Keywords:** Simple mode estimate, mixing sequence, consistency.

**2010 Mathematics Subject Classification:** 62Gxx

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## ON EFFICIENCY OF SPLIT-PLOT RESPONSE SURFACE DESIGNS WHEN SOME OBSERVATIONS ARE MISSING

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### Abstract:

In most experimental situations, some observations are lost due to unforeseen circumstances. This leads to destruction of desirable design properties like independence, normality of errors and optimality. Efficiency of completely randomized response surface designs when some observations are missing has been extensively studied in literature. However, complete randomization of experimental runs is often unrealistic in most industrial experimental situations where some factors are difficult to change. Therefore such designs are often conducted within a split-plot structure and their performance thus depends on relative magnitude( $d$ ) of model's whole-plot and subplot variance components. Split-plot central composite designs (CCD) consist of factorial( $f$ ), whole-plot axial( $\alpha$ ), subplot axial( $\beta$ ), and center( $c$ ) points. This study examines the effect of missing pairs of observations of these points on efficiency of split-plot central composite designs in terms of trace( $A$ ), maximum prediction variance ( $G$ ), and integrated average prediction variance ( $V$ ) optimality criteria, under various values of  $d$ . Efficiency functions were formulated in terms of these criteria and efficiency of reduced designs (due to missing observations), relative to the corresponding full designs, were examined. Maximum  $A$ -efficiency losses of 19.1, 10.6, and 15.7% due to missing pairs of observations,  $ff$ ,  $\beta\beta$ , and  $f\beta$ , respectively, were observed at  $d = 0.5$ ; maximum  $G$ - and  $V$ -efficiency losses of 10.1, 0.1, 16.1, 0.1% and 0.1, 0.1, 1.1, 0.2% were observed, respectively when the pairs  $ff$ ,  $\alpha\alpha$ ,  $\beta\beta$ ,  $cc$ , were missing.  $A$ -efficiency was observed to be robust to missing  $cc$ ,  $\alpha\alpha$ ,  $\alpha c$ ,  $fc$ ,  $fa$  observations while  $G$  and  $V$ -efficiencies were each observed to be robust to missing  $\alpha\alpha$ . The study revealed that as  $d$  increases, the observed losses in efficiency of these designs become insignificant.

**Keywords:** Missing observations, Split-plot central composite design, Efficiency

### 2010 Mathematics Subject Classification:

### References

- [1]
- [2]
- [3]
- [4]

## LOCALLY CORRECT CONFIDENCE INTERVALS FOR A BINOMIAL PROPORTION

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### **Abstract:**

Well-recommended methods of forming confidence intervals for a binomial proportion give interval estimates that do not actually meet the definition of a confidence interval, in that their coverages are sometimes lower than the nominal confidence level. As the definition of a confidence interval is not being adhered to, another criterion for forming interval estimates for a binomial proportion is needed.

In this work we suggest a new criterion; methods which meet the criterion are said to yield *locally correct confidence intervals*. We propose a method that yields such intervals and show that its intervals have an appreciably smaller average length.

**Keywords:** coverage; discrete distribution; shortest interval.

## SKEWED-KOTZ DISTRIBUTION WITH APPLICATION TO FINANCIAL STOCK RETURNS

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**Abstract:** This paper introduced a five-parameters skewed-Kotz (SK) distribution, that may be viewed as a generalized skewed-T distribution. Its mathematical properties are investigated, and parameters are estimated using the maximum likelihood method. The usefulness of this new distribution has been illustrated by deriving explicit formulae for the value-at-risk (VaR) and the average value-at-risk (AVaR). The obtained results are clearly generalizations of those that were established earlier by Dokov et al. (J.Appl Funct. Anal. 3(1):189-208, 2008). Simulation studies have been conducted and showed the accuracy of the VaR and AVaR computations. Furthermore, an application on financial returns of the Universal Health Services stock provided evidence that the SK distribution better fits the empirical distribution than both normal and skewed-T distributions. The empirical study revealed the suitability of the SK distribution, specially for modelling data that fall within a small range, with a high excess kurtosis.

**Keywords:** skewed-Kotz distribution, financial returns, value-at-risk.

**2010 Mathematics Subject Classification:** 62E15, 62P05

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## ASSESSMENT PRACTICES IN THE UNDERGRADUATE STATISTICS PROGRAMS IN THE ARAB WORLD

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### **Abstract:**

Assessing student learning is a vital component that should be given a high priority in higher education institutions worldwide. Evidence of learning assessment is necessary for effective program review and accreditation. The purpose of this study is to investigate the current state of assessment practices in undergraduate statistics programs in the Arab universities. A survey was sent to the chairs/coordinators of over 70 undergraduate statistics programs in Arab universities to gauge information about the adopted program learning assessment process and associated assessment activities. The program learning outcomes (PLOs) were benchmarked against the 2014 American Statistical Association (ASA) curriculum guidelines [1] and the reported assessment practices were evaluated based on the best international assessment practices. The results indicated that around two-thirds of the surveyed programs have written PLOs. Several weaknesses; however, were observed in the coverage and the wording of the PLOs and the methods adopted for assessment.

**Keywords:** undergraduate statistics education, Arab World, Program Learning outcomes assessment.

**2010 Mathematics Subject Classification:** 62

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## MODIFICATION OF GENERALIZED SPACE-TIME AUTOREGRESSIVE MODEL FOR PREDICTION MONTHLY INCIDENCE RATE IN BANYUMAS REGENCY, INDONESIA

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**Abstract:** Generalized space-time autoregressive (GSTAR) model [1] is one of space-time model which is used quite often for analyzing or forecasting spatio-temporal data, such as GDP data [2] or tourist arrivals data [3]. However, the model is rarely used in epidemiology research. In this research, we propose modification of GSTAR model for prediction monthly dengue incidence rate in urban area in Banyumas regency covering 4 sub-districts, i.e West Purwokerto, East Purwokerto, North Purwokerto, and South Purwokerto. Data from 2013 to 2018 were obtained from Banyumas Health Office. The model modification will cover seasonal factor, presence of outliers and dependent errors. In this talk, we present preliminary results of our research that is ordinary GSTAR model building, included model identification, parameter estimation, and diagnostic checking to identify unusual model behavior.

**Keywords:** Space-time model, dengue, autoregressive.

**2010 Mathematics Subject Classification:** 62-07, 37M10, 92D30

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# MULTIPLE REGRESSION MODEL TO EXAMINE THE INCIDENCE OF GOVERNMENT EXPENDITURE ON NEONATAL MORTALITY IN NIGERIA

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## Abstract:

The study examined the incidence and accessed the impact of governments' expenditure on the incidence of Neonatal Mortality in Nigeria. This was tested with use of multiple regression analysis of Neonatal Mortality against the total recurrent expenditure ( $X_1$ ) and total capital expenditure ( $X_2$ ) of the federal government of Nigeria allocated to the Health sector taking into consideration the available dataset from 1990 to 2017. The Multiple regression model derived from the dataset is  $Y = 50.9000 - 0.06023X_1 - 0.0535X_2$ . The null hypothesis test was rejected, which was an indication that the study has statistically significantly identified that both the Recurrent Expenditure and Capital Expenditure contributes greatly to the decline in Neonatal Mortality in Nigeria

**Keywords:** - Regression; Under-five; Recurrent; Endogenous; Antenatal; and Neonatal.

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## ZERO-INFLATED MODELS APPLICATION TO MATERNAL MORTALITY DATA

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### **Abstract:**

Health sector data are mostly count with many zeros and thus prompt the extension of the Poisson regression model to accommodate the zero-inflation and overdispersion. We explored all different extension of the Poisson model based on mixture models. Maximum likelihood expression for all models were derived and applied to maternal mortality data from fifty six health facilities in four regions of Ghana. The overall best model revealed that maternal mortality is based on the number of referrals (into and out) of the hospital facility, number of antenatal visits exceeding four, number of obstetric cases with HIV/AIDS, number of midwives and medical doctors at the facility.

**Key words:** Poisson regression model, overdispersion, zero-inflation.

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# Topology and Geometry

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## THE SPACE OF TWO-GENERATOR KLEINIAN GROUPS

Hala Alaqad

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**Abstract:** Let  $\text{Möb}(\overline{\mathbb{C}})$  be the group of orientation preserving Möbius transformations on the extended complex plane  $\overline{\mathbb{C}}$ . Define the following triple of complex *parameters* for each two-generator subgroup  $\langle f, g \rangle$  of  $\text{Möb}(\overline{\mathbb{C}})$  :

$$\gamma(f, g) = \text{tr}([f, g]) - 2, \beta(f) = \text{tr}^2(f) - 4, \text{ and } \beta(g) = \text{tr}^2(g) - 4,$$

where the trace of commutator  $\text{tr}([f, g]) = \text{tr}(fgf^{-1}g^{-1})$ . A *Kleinian* group is a discrete and non-elementary subgroup of  $\text{Möb}(\overline{\mathbb{C}})$ . Every two-generator Kleinian group  $\langle f, g \rangle$  is determined uniquely up to conjugacy by its triple of parameters  $(\gamma(f, g), \beta(f), \beta(g))$ . Thus, we can view the space of two-generator Kleinian groups:

$$\{\langle f, g \rangle : f, g \in \text{Möb}(\overline{\mathbb{C}})\}$$

as a subset of the three complex dimensional space  $\mathbb{C}^3$ , via the map

$$\langle f, g \rangle \longmapsto (\gamma(f, g), \beta(f), \beta(g)).$$

A sequence of two-generator subgroups  $\{\langle f_j, g_j \rangle\}$  of  $\text{Möb}(\overline{\mathbb{C}})$  *converges algebraically* to a two-generator subgroup  $\langle f, g \rangle$  of  $\text{Möb}(\overline{\mathbb{C}})$  if two sequences of generators  $\{f_j\}$  and  $\{g_j\}$  converge to  $f$  and  $g \in \text{Möb}(\overline{\mathbb{C}})$ , respectively. A fundamental result concerning space of two-generator Kleinian groups is that it is closed in the topology of algebraic convergence. We will present the recent progress including that the subspace

$$\mathcal{D} = \{(\gamma, \beta, \beta') \in \mathbb{C}^3 : \gamma = \gamma(f, g), \beta = \beta(f), \beta' = \beta(g)\},$$

is closed in three complex dimensional space  $\mathbb{C}^3$ , where  $(\gamma(f, g), \beta(f), \beta(g))$  is the triple of parameters of each two-generator Kleinian group  $\langle f, g \rangle$ . This is the joint work with Jianhua Gong and Gaven Martin, and the research project is supported by UAE University research grant UPAR G00002670.

**Keywords:** Space of Kleinian groups, triple of parameters, and algebraic convergence.

**2010 Mathematics Subject Classification:** 20H10, 22E40, 53A35

## SOME OPERATORS AND LIMIT POINTS OF A SOFT SET USING SOFT SOMEWHERE DENSE SETS

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### Abstract:

In the current work, we continue studying new notions and properties of soft somewhere dense and soft cs-dense sets. We introduce the concepts of S-interior, S-closure, S-boundary and S-limit soft points of a soft set. We investigate the relationships between them with the help of illustrative examples and discuss the role of strongly soft hyperconnected spaces in obtaining new results. Also, we prove that the operators of S-interior and S-closure are preserved under the finite soft product space. Moreover, we conclude some interrelations of them which are kept between soft topology and its parametric topologies.

**Keywords:** somewhere dense set, S-interior soft point, S-limit soft point.

**2010 Mathematics Subject Classification:** 54A05, 54B10, 03E72

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## ON FOX-TRAPEZOIDAL CONJECTURE FOR CLOSED 3-BRAIDS

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)

### Abstract:

In 1962, Fox conjectured that the coefficients of the Alexander polynomial of an alternating knot have a curious behavior. They form a trapezoid i.e., the coefficients first increase, then stabilize and finally decrease in a symmetric way. This challenging conjecture has been proved for several classes of alternating knots such as the two-bridge knots, alternating algebraic knots, alternating knots of genus 2 and alternating stable knots [1-4]. In this paper, we focus on links with braid index 3. It is well known that 3-braids have been completely classified, up to conjugation, by Murasugi [5]. Based on this classification, we prove the trapezoidal conjecture for some special cases of alternating 3-braids links. Our proof is elementary and is based on the Burau representation of the braid group.

**Keywords:** Alexander polynomial, Alternating knots, 3-braids.

**2010 Mathematics Subject Classification:** 57M25

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## COMBINATORIAL INVARIANTS OF STRATIFIED SPACES

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### **Abstract:**

The abstract is not to exceed one page.

**Keywords:** Euler characteristics, locally compact spaces, Grothendieck ring.

### **2010 Mathematics Subject Classification:**

Stratifications provide a way, in both geometry and topology, to simplify the complexity of spaces. We explain how we can define algebraic rings out of stratified data, and then homomorphisms from these rings to other commutative rings which are generalizations of the Euler characteristic. We use this method to derive interesting formulas of topological Euler characteristics of various functorial constructions on a vast collection of spaces.

## RATIONAL HOMOTOPY METHODS IN GRAPH THEORY

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### Abstract:

Inspired by the fundamental work of Luchuga and Murillo [1] who established a connection between graph theory and rational homotopy theory, these paper defines new algebraic invariants for a non-oriented, simple, connected and finite graph  $G$  namely the rational cohomology  $H^*(G, Q)$ , the Lusternik-Schnirelmann category  $cat(G)$ , the cohomology Euler-Poincaré characteristic  $\chi_G$ , the Koszul-Poincare series  $U_G(z)$  and the formal dimension  $fd(G)$ . Moreover we compute those invariants by exploiting some deep well known theorems from rational homotopy theory.

**Keywords:** Colorable graphs, Rational homotopy theory, Elliptic spaces.

**2010 Mathematics Subject Classification:** 05C15, 55P62

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