Five points on a sphere.

Submission deadline: October 28$^{\text{th}}$ 2026

Given any five distinct points on a sphere, show that there is always a way to slice the sphere into two equal hemispheres such that one of the closed hemispheres contains four out of the five points.

The problem was correctly solved by :

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Solution: Pick any two of the five points. There is a great circle passing through the 2 points, (if they are polar opposites, then there are infinitely many great circles containing the two points, so we just pick one). Cutting along this great circle, we obtain two hemispheres. By the Pigeonhole Principle, at least one of the two closed hemispheres must contain two of the remaining three points. With the two points on its boundary, that hemisphere then contains 4 of the 5 original points.