# Bisector

**Submission deadline: December $28^{\text{th}}$ 2023**

Let $a, b$ be lengths of two sides of a triangle and assume that the length of the remaining side is 2023. Further let $d$ be the length of the bisector of the angle opposite to the side with length 2023, terminated on the side. Express $d$ using $a$ and $b$.

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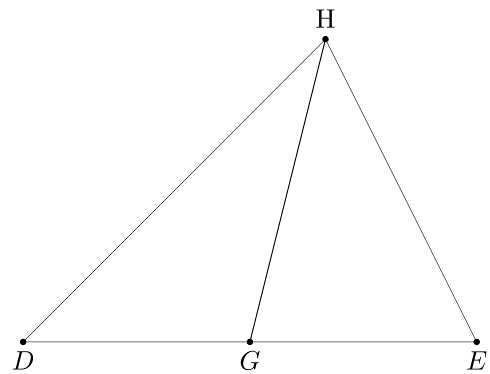
Discussion:




Let $a = |HD|, b = |HE|, d = |HG|, c_1 = |DG|$ and $c_2 = |GE|$. Then,

$$\frac{c_1}{c_2} = \frac{a}{b} \text{ and } c_1 + c_2 = 2023.$$

Solving the two equations above we get, $c_1 = \dfrac{2023a}{a+b}$ and $c_2 = \dfrac{2023b}{a+b}$. Denote the angle $DHG$ by $\alpha$. Using the Law of Cosines we get,

$$c_1^2 = a^2 + d^2 - 2ad\cos(\alpha) \text{ and } c_2^2 = b^2 + d^2 - 2bd\cos(\alpha)$$

Solve each of the equations above for $d\cos(\alpha)$ and it is easy to see that

$$\frac{c_1^2 - a^2 - d^2}{a} = \frac{c_2^2 - b^2 - d^2}{b}$$

Solve the equation above for $d^2$ to obtain

$$d^2 = ab\left(1 - \frac{2023^2}{(a+b)^2}\right)$$