The Last Digit

Submission deadline: February 28^{th} 2024

Prove that the last digit of 2^{2^n} is 6 for $n \geq 2$.

The problem was solved by

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Discussion:

We use mathematical induction. The assertion is clearly true for $n = 2$. Now assume that it is true up to k. We have that

$$2^{2^k} = 10m + 6$$

for some natural number m. Therefore,

$$(2^{2^k})^2 = (10m + 6)^2$$

Thus,

$$2^{2^{k+1}} = 100m^2 + 120m + 36.$$

The remainder of $100m^2 + 120m + 36$, upon dividing by 10 is 6. Hence, the assertion is true for all natural numbers not less than 2.