Interval

Submission deadline: September 28th 2023

Find all real roots of the equation

\[ \sqrt{x + 3 - 4\sqrt{x - 1}} + \sqrt{x + 8 - 6\sqrt{x - 1}} = 1 \]

The problem was solved by

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Discussion:
Let \( v = \sqrt{x - 1} \). Then, \( x = v^2 + 1 \), and the given equation becomes

\[
\sqrt{(v - 2)^2} + \sqrt{(v - 3)^2} = 1.
\]

Thus, we have

\[
|v - 2| + |v - 3| = 1.
\]

Clearly the equation above is satisfied if and only if \( v \) is in the interval \([2,3]\).
Hence, the given equation is satisfied if and only if \( x \) is in \([5, 10]\).