# Interval

Find all real roots of the equation

$$\sqrt{x + 3 - 4\sqrt{x - 1}} + \sqrt{x + 8 - 6\sqrt{x - 1}} = 1$$

The problem was solved by

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Discussion:

Let $v = \sqrt{x - 1}$. Then, $x = v^2 + 1$, and the given equation becomes

$$\sqrt{(v-2)^2} + \sqrt{(v-3)^2} = 1.$$

Thus, we have

$$|v - 2| + |v - 3| = 1.$$

Clearly the equation above is satisfied if and only if $v$ is in the interval [2,3]. Hence, the given equation is satisfied if and only if $x$ is in [5, 10].